

Embedding the Pentagon

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ABSTRACT: The Pentagon Model is an explicit supersymmetric extension of the Standard Model, which involves a new strongly-interacting $SU(5)$ gauge theory at TeV-scale energies. We show that the Pentagon can be embedded into an $SU(5) \times SU(5) \times SU(5)$ gauge group at the GUT scale. The doublet-triplet splitting problem, and proton decay compatible with experimental bounds, can be successfully addressed in this context. The simplest approach fails to provide masses for the lighter two generations of quarks and leptons; however, this problem can be solved by the addition of a pair of antisymmetric tensor fields and an axion.

KEYWORDS: GUT, Supersymmetry Phenomenology, Discrete and Finite Symmetries.

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1. Introduction

The Pentagon model [3, 4] is the simplest model of TeV scale physics, which is compatible with the hypothesis of Cosmological SUSY Breaking (CSB) [1] and with phenomenology. The original model [3] relied on a complicated singlet sector, which *might* have supported a meta-stable¹ SUSY violating state. That model contained a light, axion with relatively low decay constant. It could be made barely compatible with experiment, but only by re-introducing the strong CP problem.

The remodeled version of the Pentagon model [4] relied instead on the arguments of Intriligator Seiberg and Shih [10](ISS), that SUSY QCD with $N_F = N_C$ and a mass term had a meta-stable SUSY violating state.² There was no light axion and the singlet sector

¹Meta-stability is used in the sense of non-gravitational effective field theory. The arguments of [8] and [4] suggest that the de Sitter solution corresponding to this state is as stable as the equilibrium configuration of a finite system with a large number of degrees of freedom can ever be. “Instabilities” occur on the recurrence time scale and represent transient fluctuations into low entropy states.

²These arguments have been criticized in [11]. If we give separate mass to one pentaquark and add the coupling to singlets used in the Pentagon model, then the meta-stable state exists in one portion of the two parameter phase diagram. The question of where the phase boundary is, and whether it extends into the region of phenomenological relevance for the Pentagon model, cannot be answered by the perturbative methods of [11].

consisted of a single chiral superfield S . The model contains a scalar pseudo-Goldstone boson, the *penton*, stemming from the spontaneous breakdown of *pentabaryon number*, which is a characteristic of the ISS state. This particle probably evades all experimental bounds, but might be discovered in a re-analysis of (or further experiments on) flavor changing charged current hadron decays. If the scale at which the accidental pentabaryon number symmetry is explicitly broken is between 10^8 and 10^{10} GeV, the penton field might be responsible for both baryogenesis and dark matter [5]. We remind our readers that, like most low energy SUSY breaking models, the Pentagon does not have a SUSY neutralino dark matter candidate.

In the present paper we will *not* assume that the scale of pentabaryon number symmetry breaking is in this range. If the symmetry breaking scale for pentabaryon number takes the more natural value of the unification scale, then the penton *might* be the origin of baryogenesis, but will make a negligible contribution to the dark matter density. The more ambitious program of [5] would require us to explain the appearance of the intermediate scale, and to make sure that the physics at this scale does not lead to proton decay. We will not attempt to construct such a model in this paper. Indeed, the remodeled Pentagon has a strong CP problem, which we propose to solve with a QCD axion with large decay constant, f_a . The current (cosmological history independent) upper bound on f_a is of order $10^{14} - 10^{15}$ GeV [9], and it can easily be used as a dark matter candidate. We will find that most of our unification scale models require us to introduce the axion for a rather different task: the cancellation of discrete anomalies. Thus, the scenario suggested by the present paper is that axions are the dark matter, while the penton might play a role in the generation of baryon asymmetry. We will call the superfield that contains the axion X . Most other phenomenological problems of generic SUSY models are resolved by the general structure of the Pentagon. However, the question of whether it predicts a consistent pattern for the electro-weak breaking scale and the super-partner spectrum depends on strong coupling physics and does not have a definitive answer at this time. The model contains new degrees of freedom at the TeV scale so it is not obvious that it has a *little hierarchy problem*[6].

At the one loop level, the Pentagon model is compatible with coupling unification, with a GUT scale coupling that is barely perturbative. Dimension 6 proton decay is probably within reach of planned experiments. The purpose of the present paper is to see whether the Pentagon model can indeed be embedded in a unified model. It is the authors' opinion that the most plausible explanation of the discrepancy between the unification and Planck scales is that proposed by Witten [15] in the context of the Hořava-Witten [16] strongly coupled heterotic string. In this sort of scenario, quantum gravitational corrections to the four dimensional effective field theory of matter³ are expected to be scaled by the unification scale $M_U \sim 2 \times 10^{16}$ GeV.

It is therefore not strictly correct to use effective field theory to describe gauge unification. In this paper, we do this as a temporary stopgap measure. It is highly probable

³This term is used to distinguish fields whose origin is on a brane, from bulk fields like the four dimensional graviton.

that none of the currently understood supersymmetric string solutions corresponds to the zero cosmological constant (c.c.) limit of CSB [1]. It is absolutely certain that at most one of them does. To make progress without making a commitment to a particular string theory model, we resort to effective field theory, but do not neglect higher order terms in the superpotential.⁴

Our strategy will be to find a GUT model whose spectrum below the GUT scale consists of precisely the fields of the Pentagon model. This requires us to solve the doublet-triplet splitting problem, and to find an origin of the $SU(3,2,1)$ singlet field of the Pentagon (which cannot be a singlet of the unified group). To do this, we employ the strategy of [12], realizing the standard model as part of the diagonal subgroup of an $SU(5) \times SU(5)$ gauge group, broken by fields in the $(5, \bar{5})$ and $(\bar{5}, 5)$. We will also require that the theory contains an exact Z_4 R symmetry, which is preserved by the vacuum state. In the philosophy of CSB [1] this symmetry of the effective field theory is explicitly broken by interactions of the gravitino with the cosmological horizon [2] and the symmetry breaking terms vanish like a power of the c.c. The leading symmetry breaking term induces spontaneous SUSY breaking and gives a gravitino mass of order $\Lambda^{1/4}$. In the Pentagon model this is the ISS mass term $m_{\text{ISS}} P_a^i \tilde{P}_i^a$. In previous discussions it has been assumed that all other explicit R breaking was a higher power of the c.c., and therefore negligible.

In the next section, we construct what we believe is the simplest model realizing the goals of the above paragraph. However, in section 4 we find that we cannot reproduce the exact structure of the Pentagon model, in the sense that we cannot realize the Z_4 charge assignments that were used in [4]. This is a consequence of the intricate requirements imposed by anomaly cancellation, both for the gauge group and for the Z_4 symmetry. As a consequence we find that we cannot choose the charges to both eliminate dangerous operators which could lead to proton decay, and allow full rank mass matrices for quarks and leptons. In [4], the Z_4 was chosen generation blind, but this is impossible in the model we construct. The problems remain even if we try to impose other anomaly free discrete symmetries, or use a Green-Schwarz mechanism involving the field X to cancel some of the discrete anomalies.

Eventually, we traced the problem back to the fact that our model had an odd number of chiral fields in each of the $SU(5)$ groups. The simplest way to solve it would be to add an additional $(\bar{5}, 1) \oplus (1, 5)$ to the model, but this leaves over too many massless low energy fields. This can be remedied if we add a $(\bar{10}, 1) \oplus (1, 10)$. We then obtain a model whose low energy spectrum and Z_4 charge assignments agree precisely with the Pentagon model. To cancel discrete anomalies we have to resort to a Green-Schwarz mechanism involving X . We discuss this model in section 5. In the conclusions, we make some comments about the implementation of the Froggatt-Nielson mechanism in this model, and about the possibility of allowing R parity violating couplings that might be useful for resolving the little hierarchy problem [7]. Before concluding this introduction, we want to emphasize for clarity that, although the Pentagon model was motivated by the highly speculative idea

⁴We will be searching for supersymmetric vacua, so we will not need to say anything about the Kahler potential.

of CSB, it is just a low energy effective field theory. The only way in which CSB affects any of the analysis of the Pentagon model is through an *a priori* constraint on the size of the mass parameter m_{ISS} . For readers who prefer to ignore CSB, one can imagine that this parameter is determined by *retro-fitting*[17]. That is one assumes that it arises from a non-renormalizable coupling to e.g. the squared field strength of a pure supersymmetric gauge theory with scale Λ_H . By playing with Λ_H one can obtain a value of m_{ISS} in a phenomenologically acceptable range.

2. A minimal model

2.1 GUT breaking fields and R-charge assignments

Following [12], we introduce an $SU_1(5) \times SU_2(5)$ gauge group at the GUT scale, in addition to the $SU_P(5)$ of the Pentagon. The Standard Model matter fields (three $\bar{5}$'s and three 10's) can each reside in either of these $SU(5)$'s (transforming as singlets under the other) subject to the constraint that there are no anomalies in the gauge symmetry. The SM Higgs fields come from the doublet components of H_u which transforms as a $(5, 1)$, and H_d which transforms as a $(1, \bar{5})$. We add two bifundamental fields Φ_1 and Φ_2 that transform as $(5, \bar{5})$ under the gauge group, and 2 bifundamental fields $\tilde{\Phi}_1$ and $\tilde{\Phi}_2$ that transform as $(\bar{5}, 5)$'s. These fields will be responsible for breaking the GUT-scale gauge group. We adopt the method for solving the doublet-triplet splitting problem discussed in [12]. Assume a SUSic minimum at the following VEVs for the Φ fields:

$$\langle \Phi_1 \rangle = \begin{pmatrix} v_1 & 0 & 0 & 0 & 0 \\ 0 & v_1 & 0 & 0 & 0 \\ 0 & 0 & v_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \langle \tilde{\Phi}_1 \rangle = \begin{pmatrix} \tilde{v}_1 & 0 & 0 & 0 & 0 \\ 0 & \tilde{v}_1 & 0 & 0 & 0 \\ 0 & 0 & \tilde{v}_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\langle \Phi_2 \rangle = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & v_2 & 0 \\ 0 & 0 & 0 & 0 & v_2 \end{pmatrix} \quad \langle \tilde{\Phi}_2 \rangle = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \tilde{v}_2 & 0 \\ 0 & 0 & 0 & 0 & \tilde{v}_2 \end{pmatrix}$$

If there is a discrete symmetry, which allows a coupling of the Higgs fields to Φ_1 or $\tilde{\Phi}_1$ but not Φ_2 or $\tilde{\Phi}_2$, the triplet Higgs will attain a GUT-scale mass after symmetry breaking while the doublet Higgs will remain massless. The Φ_2 fields must get VEVs in the doublet sector in order to fully break the $SU(5) \times SU(5)$ symmetry down to $SU(3) \times SU(2) \times U(1)$. If only the Φ_1 fields got non-zero VEVs, there would still be an unbroken $SU(2) \times SU(2)$ subgroup.

We also need pentaquark fields, and a field that can couple to the pentaquarks in such a way that a massless $SU(3, 2, 1)$ singlet remains below the GUT scale, which can play the role of the S field in the Pentagon model. For this we will introduce a high energy S

field which transforms as a $5 \times \bar{5}$ in $SU_{1,2}(5)$, and a T field which transforms as a 24 in $SU_{1,2}(5)$. (Henceforth, we will refer to the low energy field of the Pentagon Model, which survives symmetry breaking, as s .) The plethora of GUT scale fields is needed in order to ensure that the post GUT spectrum be precisely that of the Pentagon model. The pentaquark fields P and \tilde{P} must both be in the same $SU(5)$ group to avoid anomalies, and this must be the same group in which S transforms in order for there to be a coupling $SP\tilde{P}$. They are (anti-)fundamentals in $SU_{1,2}(5)$ and in the Pentagon gauge group, but singlets in $SU_{2,1}(5)$. The choice of which $SU(5)$ group the T field transforms under defines two classes of models, which we will later distinguish with a two valued parameter $p = 0, 1$. Note that the distinction between the two $SU(5)$ groups is that the high energy avatar of H_u transforms under $SU_1(5)$.

Finally, we have to impose a Z_4 R symmetry, to match that of the low energy Pentagon model. It is tempting to imagine that this R symmetry also plays the role of forbidding the unwanted couplings between the Higgs fields and the Φ fields. The low energy R symmetry, Z'_4 may be a combination of the high energy Z_4 with elements of the spontaneously broken GUT group.

Given the VEVs above for the Φ fields and the following two requirements:

1. The low energy theory contains a leftover Z'_4 R-symmetry after the high energy GUT group is spontaneously broken.
2. The $SU(2)$ block of components of Φ_1 at low energies have R-charge 2 under the Z'_4 .⁵

there is a unique assignment of the R-charges for the Φ fields. There can be a low-energy Z'_4 preserved only if there is some combination of $SU(5)$ transformations combined with the high-energy Z_4 transformation that preserves the VEVs of the Φ fields (in addition, the VEVs of S and T must be preserved, but we will not impose that just yet). This can be accomplished by a simultaneous anti-diagonal $U(1)$ (hypercharge) rotation in each of the $SU(5)$ groups.⁶

$$\begin{aligned} \langle \Phi_1 \rangle &\rightarrow \exp\left(\frac{2\pi i}{4}q_1\right) \exp(2i\alpha)\langle \Phi_1 \rangle \\ \langle \tilde{\Phi}_1 \rangle &\rightarrow \exp\left(\frac{2\pi i}{4}\tilde{q}_1\right) \exp(-2i\alpha)\langle \tilde{\Phi}_1 \rangle \\ \langle \Phi_2 \rangle &\rightarrow \exp\left(\frac{2\pi i}{4}q_2\right) \exp(-3i\alpha)\langle \Phi_2 \rangle \\ \langle \tilde{\Phi}_2 \rangle &\rightarrow \exp\left(\frac{2\pi i}{4}\tilde{q}_2\right) \exp(3i\alpha)\langle \tilde{\Phi}_2 \rangle \end{aligned}$$

⁵We make this assignment to ensure that the diagonal (singlet) part of it can mix with the other massless singlet fields to form the s field of the Pentagon model. s must have R-charge 2 and needs to contain a piece of Φ_1 in order to have a coupling to the Higgs.

⁶A diagonal rotation is not useful at present since it is part of the unbroken symmetry and does not affect the Φ VEVs. Combining the specific antidiagonal transformation we find in this section with different diagonal transformations yields a class of gauge-equivalent low energy R-symmetries.

where α is the angle of an anti-diagonal U(1) rotation in each of the SU(5)'s ($e^{i\alpha}$ in the SU(3) subgroup of $SU_1(5)$, and $e^{-i\alpha}$ in the SU(3) subgroup of $SU_2(5)$, with $e^{-\frac{3}{2}i\alpha}$ and $e^{\frac{3}{2}i\alpha}$ for the corresponding SU(2) subgroups). The constraint that the Φ VEVs be preserved can then be written:

$$\begin{aligned} \frac{q_1}{4} + 2\alpha' &= n \\ \frac{\tilde{q}_1}{4} - 2\alpha' &= m \\ \frac{q_2}{4} - 3\alpha' &= l \\ \frac{\tilde{q}_2}{4} + 3\alpha' &= r \end{aligned}$$

where $\alpha' \equiv \frac{\alpha}{2\pi}$, and n, m, l, r are integers. Combining the first two equations requires that $\tilde{q}_1 = -q_1$, and the last two equations imply that $\tilde{q}_2 = -q_2$. In other words, the tilded fields must have R-charge opposite that of the corresponding untilded fields. Since the SU(2) block of Φ_1 transforms as $\exp(\frac{2\pi i}{4}q_1)\exp(-3i\alpha)$, the constraint coming from condition 2 above is:

$$\frac{q_1}{4} - 3\alpha' = 1/2 + j \tag{2.1}$$

where j is another independent integer. Combining this with the first and third equations from constraint 2 implies that $q_1 = 0 \pmod{4}$, $q_2 = 2 \pmod{4}$, and $\alpha' = \frac{1}{2} \pmod{1}$ (or $\alpha = \pi$). Therefore, the only assignment of high-energy R-charges that is compatible with the two requirements is $R(\Phi_1) = 0$, $R(\tilde{\Phi}_1) = 0$, $R(\Phi_2) = 2$, $R(\tilde{\Phi}_2) = 2$.

The U(1) transformation which yields Z'_4 when combined with Z_4 is given by:

$$G = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & i & 0 \\ 0 & 0 & 0 & 0 & i \end{pmatrix} \tag{2.2}$$

where the transformation we need is ‘‘anti-diagonal’’ in the sense that we combine G acting on $SU_1(5)$ with G^\dagger acting on $SU_2(5)$. The fields then transform like $\Phi_i \rightarrow G\Phi_iG$, $\tilde{\Phi}_i \rightarrow G^\dagger\tilde{\Phi}_iG^\dagger$, $S \rightarrow GSG^\dagger$, and $T \rightarrow GTG^\dagger$. The components in the SU(3) block of all of the Φ fields are even under the U(1) rotation, so they have the same Z'_4 charges as their Z_4 charges. However, the components in the SU(2) block of all of the Φ fields are odd under the U(1) rotation, so their Z'_4 charges are opposite to their Z_4 charges. The components in the off-diagonal blocks of the Φ_i fields transform as $\Phi_i^{(3,2)} \rightarrow -i\Phi_i^{(3,2)}$ (and the same for $\tilde{\Phi}_i^{(3,2)}$, where the superscript is referring to their transformation properties under the appropriate SU(3) \times SU(2) subgroup), so their Z'_4 R-charges are their Z_4 charges minus 1. The $\tilde{\Phi}_i^{(3,2)}$ and $\tilde{\Phi}_i^{(3,2)}$ components transform oppositely to this, so their Z'_4 charges are their Z_4 charge plus 1.

For the S and T fields, the on-diagonal blocks are both invariant under the hypercharge transformation, so their R-charges are unchanged after the symmetry is broken. Assuming the S and T fields are in $SU_1(5)$, the R-charges of the (3, 2) (lower-left) blocks of the S and

	$SU(5) \times SU(5)$	Z_4	$(8, 1) Z'_4$	$(1, 3) Z'_4$	$(3, 2) Z'_4$	$(\bar{3}, 2) Z'_4$
Φ_1	$(5, \bar{5})$	0	0	2	3	3
$\tilde{\Phi}_1$	$(\bar{5}, 5)$	0	0	2	1	1
Φ_2	$(5, \bar{5})$	2	2	0	1	1
$\tilde{\Phi}_2$	$(\bar{5}, 5)$	2	2	0	3	3
S	$(5 \times \bar{5}, 1)$	2	2	2	1*	3*
T	$(24, 1)$	0	0	0	3*	1*

Table 1: High and low energy R-charges of GUT-breaking fields.

T fields are decreased by 1, whereas the R-charges of the $(\bar{3}, 2)$ (upper-right) blocks of the S and T fields are increased by 1. This assignment is reversed if one or both of the fields is in $SU_2(5)$. Note that the $(3, 2)$ numbers here refer to charges under the $SU(3) \times SU(2)$ subgroup of $SU_1(5)$, whereas in the previous paragraph (for the Φ 's) they are referring to a product of the $SU(3)$ subgroup from one $SU(5)$ and the $SU(2)$ subgroup from the other $SU(5)$.

In order to preserve the R-symmetry, the VEVs of the S and T fields must also be invariant under the Z'_4 transformation. The on-diagonal blocks of S and T are already invariant, but the off-diagonal blocks are not. Therefore, the off-diagonal blocks must have zero VEVs. In order to have a coupling between S and the pentaquark fields, we need the entire VEV of S to be zero to avoid giving the pentaquarks a GUT scale mass. Therefore, the only allowed non-zero VEVs for the S and T fields are in the diagonal blocks of T . After constructing the superpotential, we will need to verify that it has a minimum where this is the case.

Table 1 summarizes the Φ , S , and T fields, their Z_4 charges, and the low-energy Z'_4 charges of the different components. If the S is a $(1, 5 \times \bar{5})$ or the T is a $(1, 24)$ then the 1's and 3's (marked by *'s) in the corresponding row are reversed. There are also $(1, 1)$ singlet components in all of the fields, which are not listed in the table. There are two in each of the Φ fields: one is the trace of the $SU(3)$ block and has the same R-charge as the $(8, 1)$ components, and the other is the trace of the $SU(2)$ block and has the same R-charge as the $(3, 1)$ components. Similarly, there are also two $(1, 1)$ components in the S which have the same R-charge as the rest of the diagonal blocks. And finally, there is one $(1, 1)$ component in the T : the component proportional to the generator of $SU(5)$ responsible for $U(1)$ hypercharge transformations in the usual embedding of $SU(3) \times SU(2) \times U(1)$. In all of these cases, the Z'_4 charge of the singlets is just the same as the corresponding $(8, 1)$ or $(3, 1)$ components in the table above.

The low-energy R-charges of the matter fields of the Standard Model, Higgs fields, and pentaquarks are also different from their high-energy R-charges. Both of the Higgs doublets end up with their low-energy R-charge increased by 1. Since there is a high-energy coupling $\tilde{\Phi}_1 H_u H_d$ (this is the term which gives the triplet Higgses a GUT-scale mass when the VEV of Φ_1 is plugged in), the sum of the Higgs Z_4 charges must be 2. At low energies (in the Pentagon model) the R-charge of both Higgs fields must be opposite in order to couple to

the low-energy S field. This works out automatically, since the symmetry breaking changes their total R-charge by 2.

The pentaquarks are more complicated, since none of their components get masses after the breaking. Different components of the pentaquarks have different R-charges at low energy. However, this is equivalent to the R-symmetry described in the original Pentagon model in which there is no distinction between the R-charge of doublet and triplet components. The argument goes as follows: The R-charges of the pentaquark fields start out summing to zero, and when they get broken down they must end up summing to zero. This is automatically the case for both the doublet and triplet components, since the change in R-charge upon breaking is opposite for a $(5, 1)$ as it is for a $(\bar{5}, 1)$. Because P and \tilde{P} are the only fields charged under the *Pentagon* $SU(5)$, they only show up in pentabaryon and pentameson combinations in the low-energy superpotential. $P\tilde{P}$ still has R-charge 0, and $\det[PPPPP]$ (and similarly $\det[\tilde{P}\tilde{P}\tilde{P}\tilde{P}\tilde{P}]$) is a product of 2 doublet P's and 3 triplet P's that adds up to R-charge 0 at low energies as well. Consequently, all of the terms involving P and \tilde{P} in the original Pentagon model are present and no additional terms are present. In other words, the Z_4 R-symmetry described here differs from the one in the original Pentagon model only by a transformation which is already a symmetry of the superpotential—in particular, a $U(1)$ hypercharge transformation. Instead of using the anti-diagonal transformation G to relate the high and low energy R-symmetries, we could have chosen to perform the transformation twice in one $SU(5)$ (leaving the pentaquark fields in the other $SU(5)$ invariant). This transformation differs from G only by an element of the unbroken (diagonal) $U(1)$.

The Standard Model matter fields break down into $\bar{5} = (\bar{3}, 1) \oplus (1, 2)$ (\bar{D} and L), and $10 = (3, 2) \oplus (\bar{3}, 1) \oplus (1, 1)$ (Q , \bar{U} , and \bar{E}). In the original formulation of the Pentagon model, it was assumed that all 3 generations of \bar{D} had R-charge $2 + 3R(L) - R(H_d)$. However, this is inconsistent with our present description of L and \bar{D} as coming from a $\bar{5}$ field with a single R-charge at high energies. After GUT breaking, the R-charge of L changes by 1, whereas the R-charge of \bar{D} changes by 2. Therefore, for a particular generation one of them must have an odd R-charge and the other an even R-charge. Furthermore, in our present setup anomaly cancellation requires us to place different matter fields in different $SU(5)$ groups, which makes it necessary to drop the assumption made in the original Pentagon model that all three generations have the same R-charge at low energies. This also suggests that the high-energy R-charges may be different for each generation. We will consider all possibilities for both the high energy R-charges and the high energy $SU_1(5) \times SU_2(5)$ quantum numbers of the Standard Model matter fields. In order to stay within experimental bounds for lepton and baryon violation, it will also be necessary to add at least one additional discrete symmetry to the high energy model. Table 2 summarizes the way the R-charges of the Standard Model fields change after symmetry breaking. The value of each R-charge is abbreviated here by the corresponding field (a convention we will adhere to from now on in this paper). Depending on the generation, each field of a given type may transform either in $SU_1(5)$ or in $SU_2(5)$ (each choice corresponding to a column in the table), except for the Higgs fields whose transformation properties are fixed by our choice $SU_1(5)$ as the group under which the H_u transforms.

	$SU_1(5)$	$SU_2(5)$
L	$\mathcal{D} - 1$	$\mathcal{D} + 1$
\bar{D}	$\mathcal{D} + 2$	$\mathcal{D} + 2$
Q	$\mathcal{U} - 1$	$\mathcal{U} + 1$
\bar{U}	\mathcal{U}	\mathcal{U}
\bar{E}	$\mathcal{U} + 2$	$\mathcal{U} + 2$
h_u	$H_u + 1$	
h_d		$H_d + 1$

Table 2: High and low energy R-charges of Standard Model fields.

Knowing the R-charges of all the low energy field components allows us to explain our choice of high energy fields. The Φ fields exist at the GUT scale to provide the mechanism for breaking to the $SU(3) \times SU(2) \times U(1)$ of the Standard Model and give the triplet Higgs a mass, but they are not present in the Pentagon model and so must gain a mass. However, the model does have a singlet (s), which couples to both the Pentaquarks and the Higgs doublets. This field must originate from fields at the GUT scale with the same couplings but remain massless at low energies. $\tilde{\Phi}_1$ couples to the Higgs, but cannot couple to the Pentaquarks because it would give them a mass; this is why it was necessary to introduce a field with zero VEV (S) with such a coupling. However, the Pentaquarks both transform in a single $SU(5)$ while the Higgs must couple to fields that transform under both groups. Therefore, the Pentagon singlet s must be a massless linear combination of the singlet components of the high energy fields, and must include the singlets of S as well as the $SU(2)$ block singlet of Φ_1 (at low energies the s need only couple to the Higgs doublet). All other components of the high energy fields must acquire a mass to prohibit them from appearing in the low energy model.

The inclusion of $S \in (5 \times \bar{5}, 1)$ and $T \in (24, 1)$ in the high energy model is necessary to provide mass to all unwanted fields. In total, $S, T, \Phi_1, \tilde{\Phi}_1, \Phi_2, \tilde{\Phi}_2$ comprise 149 components; 148 must acquire mass while one remains massless. Finding exactly one zero eigenvalue by diagonalizing a 149 row \times 149 column mass matrix is not a trivial task, but this naive approach is greatly simplified by two facts: every mass term in the low energy effective lagrangian must have R-charge 2 and must exhibit $SU(3) \times SU(2) \times U(1)$ invariance. The group structure of the theory allow us to split the mass matrix into group irreducible blocks; in particular, the eleven singlets would form an 11×11 block, the $SU(2)$ components form an 18×18 block, the $SU(3)$ components a 48×48 block, and the $(3, 2)$ and $(\bar{3}, 2)$ components would combine into two identical 36×36 blocks. Each block must have all non-zero eigenvalues except for one corresponding to an eigenvector from the singlet block with components in the direction of S and the $SU(2)$ block singlet of $\tilde{\Phi}_1$.

This is indeed the case for our model, ensured by the R-charges of the fields. In the $SU(2)$ and $SU(3)$ sectors there are an equal number of R-charge 2 and R-charge 0 components, allowing every one of these components to 'pair up' into quadratic mass terms. Thus every $(3, 1)$ and $(1, 2)$ component of the high energy fields will gain a GUT

scale mass. The argument is similar for the $(3,2)$ and $(\bar{3},2)$ components: there is exactly one R-charge 3 $(3,2)$ component for every R-charge 3 $(\bar{3},2)$ (the same is true for the R-charge 1 components), again allowing each of these fields to gain mass. This is not the case for the singlet block — there are an odd number of fields, six with R-charge 2 and five with R-charge 0, so there will inevitably be a single R-charge 2 field that cannot pair with any other field and will therefore remain massless. Furthermore, in the general case this field will be some linear combination of all of the R-charge 2 singlets, including the singlets of the S and the $SU(2)$ block singlet of $\tilde{\Phi}_1$.

The choice of GUT scale field content in our model is not unique. The requirement of obtaining one massless singlet in the low energy limit, with all the rest of the fields massive, enforces two conditions. First, there must be an odd number of singlet components, $(N + 1)/2$ of which are R-charge 2 and the rest R-charge zero. This field will in general be a linear combination of all the R-charge 2 components, and so, assuming the additional requirement of the high energy couplings $SP\tilde{P}$ and $\tilde{\Phi}_1 H_u H_d$ as discussed, this singlet will include components in the directions of both S and the $SU(2)$ block singlet of $\tilde{\Phi}_1$. The second condition is that the total number of fields is even, half with R-charge 2 and half with R-charge 0, so that all components other than the singlets can pair with another field and so gain mass. Our particular choice of fields is the simplest we have found, which solves the doublet-triplet splitting problem while having low energy field content identical to the Pentagon model.

3. The superpotential

Thus far we have constructed a GUT scale $SU(5) \times SU(5) \times SU(5)$ model that spontaneously breaks to the Pentagon $SU(5) \times$ Standard Model $SU(3) \times SU(2) \times U(1)$. It remains to construct the superpotential of this model. In particular, we must show that our VEVs lie at a minimum of the potential and that the mass spectrum of the low energy effective theory does in fact leave the field content of the Pentagon model. In this section we will focus on the additional ingredients that our high energy model has contributed to the Pentagon, assuming that all other fields have zero GUT scale VEVs and so do not contribute to the discussion. A more detailed consideration of the standard matter fields will be the subject of the next section.

All terms in the superpotential must obey two rules: they must be invariant under both of the $SU(5)$ symmetries (here and for the majority of the discussion we will ignore the Pentagon $SU(5)$ as all fields but the pentaquarks transform trivially under it), and the total R-charge of each term must sum to 2. Because all of the fields under consideration have either R-charge 2 ($S, \Phi_2, \tilde{\Phi}_2$) or R-charge 0 ($T, \Phi_1, \tilde{\Phi}_1$), there must be an odd number of R-charge 2 fields in every term. The $SU(5)$ invariance has a number of consequences. We will take the group structure of the fields under $SU_1(5) \times SU_2(5)$ to be $S \in (5 \times \bar{5}, 1), T \in (24, 1), \Phi_i \in (5, \bar{5}),$ and $\tilde{\Phi}_i \in (\bar{5}, 5)$. The discussion is analogous for the case that the S and/or T transform in $SU_2(5)$. Then, since only the bi-fundamentals transform under $SU_2(5)$, gauge invariance implies that there must be an even number of them in every term, with a Φ_i always paired with a $\tilde{\Phi}_j$. These combined pairs of $\Phi\tilde{\Phi}$ behave as a single

$5 \times \bar{5}$ field under $SU_1(5)$, and can either be traced over or combined with the other fields in terms that are invariant under $SU_1(5)$.

First look at terms that only involve traces over the Φ s, we will call this piece of the superpotential W_Φ ,

$$\begin{aligned}
 W_\Phi &= M(\Phi_1\tilde{\Phi}_2) + M(\Phi_2\tilde{\Phi}_1) \\
 &+ \frac{1}{M}[(\Phi_1\tilde{\Phi}_1\Phi_1\tilde{\Phi}_2) + (\Phi_1\tilde{\Phi}_1\Phi_2\tilde{\Phi}_1) + (\Phi_1\tilde{\Phi}_2\Phi_2\tilde{\Phi}_2) + (\Phi_2\tilde{\Phi}_1\Phi_2\tilde{\Phi}_2) \\
 &+ (\Phi_1\tilde{\Phi}_1)(\Phi_1\tilde{\Phi}_2) + (\Phi_1\tilde{\Phi}_1)(\Phi_2\tilde{\Phi}_1) + (\Phi_1\tilde{\Phi}_2)(\Phi_2\tilde{\Phi}_2) + (\Phi_2\tilde{\Phi}_1)(\Phi_2\tilde{\Phi}_2)] \\
 &+ \text{higher order.}
 \end{aligned}$$

We have suppressed the $SU(5)$ index structure. Terms in parenthesis imply a trace over those fields. So for example, the term $(\Phi_1\tilde{\Phi}_1\Phi_1\tilde{\Phi}_2)$ would be written explicitly as $(\Phi_1)_A^i(\tilde{\Phi}_1)_j^A(\Phi_1)_B^j(\tilde{\Phi}_2)_i^B$, whereas the term $(\Phi_1\tilde{\Phi}_1)(\Phi_1\tilde{\Phi}_2)$ would be $(\Phi_1)_A^i(\tilde{\Phi}_1)_i^A(\Phi_1)_B^j(\tilde{\Phi}_2)_j^B$. An upper index refers to the 5 and a lower index refers to the $\bar{5}$ representation, while the lower case indices refer to $SU_1(5)$ and the upper case indices to $SU_2(5)$. We have also omitted coefficients for the terms, which in general should be arbitrary.

The mass scale appearing in these equations is of order the GUT scale, which according to our hypothesis, is the scale at which we expect quantum gravitational corrections to appear. Our strategy will be to work with polynomials of minimal order, to demonstrate that we can achieve the pattern of VEVs we used in the previous section. The low order terms do not have accidental symmetries, so we expect that higher order corrections will make order one changes to the VEVs and masses, without disturbing the qualitative nature of the system or the low energy Lagrangian.

There will be other terms in W involving the determinants of the Φ fields. We will separate these onto their own:

$$\begin{aligned}
 W_{\det} &= M^{-2}(\det[\Phi_1\Phi_1\Phi_1\Phi_1\Phi_2] + \det[\Phi_1\Phi_1\Phi_2\Phi_2\Phi_2] + \det[\Phi_2\Phi_2\Phi_2\Phi_2\Phi_2] \\
 &+ \det[\tilde{\Phi}_1\tilde{\Phi}_1\tilde{\Phi}_1\tilde{\Phi}_1\tilde{\Phi}_2] + \det[\tilde{\Phi}_1\tilde{\Phi}_1\tilde{\Phi}_2\tilde{\Phi}_2\tilde{\Phi}_2] + \det[\tilde{\Phi}_2\tilde{\Phi}_2\tilde{\Phi}_2\tilde{\Phi}_2\tilde{\Phi}_2]) \\
 &+ \text{higher order.}
 \end{aligned}$$

Next consider the terms involving the S and T fields,

$$\begin{aligned}
 W_S &= (S\Phi_1\tilde{\Phi}_1) + (S\Phi_2\tilde{\Phi}_2) \\
 &+ \frac{1}{M}[(SS\Phi_1\tilde{\Phi}_2) + (SS\Phi_2\tilde{\Phi}_1) + (SS)(\Phi_1\tilde{\Phi}_2) + (SS)(\Phi_2\tilde{\Phi}_1)] \\
 &+ \frac{1}{M^2}[(S\Phi_1\tilde{\Phi}_1\Phi_1\tilde{\Phi}_1) + (S\Phi_2\tilde{\Phi}_2\Phi_2\tilde{\Phi}_2) + (S\Phi_1\tilde{\Phi}_1\Phi_2\tilde{\Phi}_2) + (S\Phi_1\tilde{\Phi}_2\Phi_2\tilde{\Phi}_1) \\
 &+ (S\Phi_2\tilde{\Phi}_2\Phi_1\tilde{\Phi}_1) + (S\Phi_1\tilde{\Phi}_2\Phi_1\tilde{\Phi}_2) + (S\Phi_2\tilde{\Phi}_1\Phi_2\tilde{\Phi}_1) + (S\Phi_2\tilde{\Phi}_1\Phi_1\tilde{\Phi}_2) \\
 &+ (S\Phi_1\tilde{\Phi}_1)(\Phi_1\tilde{\Phi}_1) + (S\Phi_2\tilde{\Phi}_2)(\Phi_2\tilde{\Phi}_2) + (S\Phi_1\tilde{\Phi}_1)(\Phi_2\tilde{\Phi}_2) + (S\Phi_1\tilde{\Phi}_2)(\Phi_2\tilde{\Phi}_1) \\
 &+ (S\Phi_2\tilde{\Phi}_2)(\Phi_1\tilde{\Phi}_1) + (S\Phi_1\tilde{\Phi}_2)(\Phi_1\tilde{\Phi}_2) + (S\Phi_2\tilde{\Phi}_1)(\Phi_2\tilde{\Phi}_1) + (S\Phi_2\tilde{\Phi}_1)(\Phi_1\tilde{\Phi}_2)] \\
 &+ \text{higher order,}
 \end{aligned}$$

$$\begin{aligned}
W_T = & (T\Phi_1\tilde{\Phi}_2) + (T\Phi_2\tilde{\Phi}_1) \\
& + \frac{1}{M}[(TT\Phi_1\tilde{\Phi}_2) + (TT\Phi_2\tilde{\Phi}_1) + (TT)(\Phi_1\tilde{\Phi}_2) + (TT)(\Phi_2\tilde{\Phi}_1)] \\
& + \frac{1}{M^2}[(T\Phi_1\tilde{\Phi}_2\Phi_1\tilde{\Phi}_1) + (T\Phi_1\tilde{\Phi}_2\Phi_2\tilde{\Phi}_2) + (T\Phi_2\tilde{\Phi}_1\Phi_1\tilde{\Phi}_1) + (T\Phi_2\tilde{\Phi}_1\Phi_2\tilde{\Phi}_2) \\
& + (T\Phi_1\tilde{\Phi}_1\Phi_1\tilde{\Phi}_2) + (T\Phi_2\tilde{\Phi}_2\Phi_1\tilde{\Phi}_2) + (T\Phi_1\tilde{\Phi}_1\Phi_2\tilde{\Phi}_1) + (T\Phi_2\tilde{\Phi}_2\Phi_2\tilde{\Phi}_1) \\
& + (T\Phi_1\tilde{\Phi}_2)(\Phi_1\tilde{\Phi}_1) + (T\Phi_1\tilde{\Phi}_2)(\Phi_2\tilde{\Phi}_2) + (T\Phi_2\tilde{\Phi}_1)(\Phi_1\tilde{\Phi}_1) + (T\Phi_2\tilde{\Phi}_1)(\Phi_2\tilde{\Phi}_2) \\
& + (T\Phi_1\tilde{\Phi}_1)(\Phi_1\tilde{\Phi}_2) + (T\Phi_2\tilde{\Phi}_2)(\Phi_1\tilde{\Phi}_2) + (T\Phi_1\tilde{\Phi}_1)(\Phi_2\tilde{\Phi}_1) + (T\Phi_2\tilde{\Phi}_2)(\Phi_2\tilde{\Phi}_1) \\
& + (TTT\Phi_1\tilde{\Phi}_2) + (TTT\Phi_2\tilde{\Phi}_1) + (TTT)(\Phi_1\tilde{\Phi}_2) + (TTT)(\Phi_2\tilde{\Phi}_1) \\
& + (TT)(T\Phi_1\tilde{\Phi}_2) + (TT)(T\Phi_2\tilde{\Phi}_1)] + \text{higher order.}
\end{aligned}$$

and

$$\begin{aligned}
W_{ST} = & M(ST) + (STT) + \frac{1}{M}[(ST\Phi_1\tilde{\Phi}_1) + (ST\Phi_2\tilde{\Phi}_2) \\
& + (TS\Phi_1\tilde{\Phi}_1) + (TS\Phi_2\tilde{\Phi}_2) + (ST)(\Phi_1\tilde{\Phi}_1) + (ST)(\Phi_2\tilde{\Phi}_2)] \\
& + \text{higher order.}
\end{aligned}$$

The total superpotential, W , will include the sum of these pieces as well as contributions from terms containing the pentaquarks and matter fields,

$$W = W_\Phi + W_{\text{det}} + W_S + W_T + W_{ST} + W_{\text{Pentagon}}.$$

Although we will not discuss in detail the content of W_{Pentagon} , we should point out a few of the key terms mentioned earlier. Most importantly, the couplings $SP\tilde{P}$ and $H_u\tilde{\phi}_1H_d$ lead to W_S of the original Pentagon model. There will also be Yukawa terms including the fields $H_u\mathcal{U}_i\mathcal{U}_j$ and $H_d\mathcal{U}_i\mathcal{D}_j$ where the \mathcal{U}, \mathcal{D} transform as $10, \bar{5}$ respectively under one of the $SU(5)$ s; these will be discussed further in the next section.

Now that we have constructed the superpotential, we should verify that the VEVs we chose in the previous section are in fact at a minimum. We can expect that this will be achieved only by satisfying a set of six constraints — there are five degrees of freedom in the VEVs, each of which should be determined by the F -equations, and a sixth constraint will restrict the coefficients of the terms in the lagrangian in a manner required by the preservation of the R symmetry.

Let us assume that the vacuum expectation values of the Φ fields have the form discussed in the previous section, and that the VEV of S is zero. The form of the VEV of T is so far undetermined, but we know from the previous section that it must be block diagonal. Furthermore, we will see presently that all off diagonal components within these blocks will have to be zero as well, that the $SU(3)$ diagonal components must all be equal to each other (the same being true for the $SU(2)$ components), and that the trace must be

zero. Thus we will assume the VEV of T has the form

$$\langle T \rangle = \begin{pmatrix} v_T & 0 & 0 & 0 & 0 \\ 0 & v_T & 0 & 0 & 0 \\ 0 & 0 & v_T & 0 & 0 \\ 0 & 0 & 0 & -3/2v_T & 0 \\ 0 & 0 & 0 & 0 & -3/2v_T \end{pmatrix}.$$

Consider first the S equation, $F_S = \partial W / \partial S = 0$. Even before inserting the VEVs, the only surviving terms are those from W_S and W_{ST} . Terms that involve the product of the VEVs of Φ_1 or $\tilde{\Phi}_1$ times Φ_2 or $\tilde{\Phi}_2$ in W_S , and those with multiple powers of S in W_{ST} , will vanish. What remains is (again, omitting the explicit index structure; a system of equations for the individual components is implied):

$$\begin{aligned} F_S = & \Phi_1 \tilde{\Phi}_1 + \Phi_2 \tilde{\Phi}_2 + \frac{1}{M^2} [\Phi_1 \tilde{\Phi}_1 \Phi_1 \tilde{\Phi}_1 + \Phi_2 \tilde{\Phi}_2 \Phi_2 \tilde{\Phi}_2 \\ & + \Phi_1 \tilde{\Phi}_1 (\Phi_1 \tilde{\Phi}_1) + \Phi_2 \tilde{\Phi}_2 (\Phi_2 \tilde{\Phi}_2) + \Phi_1 \tilde{\Phi}_1 (\Phi_2 \tilde{\Phi}_2) + \Phi_2 \tilde{\Phi}_2 (\Phi_1 \tilde{\Phi}_1)] \\ & + MT + TT + \frac{1}{M} [T \Phi_1 \tilde{\Phi}_1 + T \Phi_2 \tilde{\Phi}_2 + \\ & \Phi_1 \tilde{\Phi}_1 T + \Phi_2 \tilde{\Phi}_2 T + T (\Phi_1 \tilde{\Phi}_1) + T (\Phi_2 \tilde{\Phi}_2)] \\ & + \text{higher order.} \end{aligned}$$

We are really taking derivatives with respect to individual components of the S_j^i , so terms not in parenthesis should be read as $(\Phi_1)_A^j (\tilde{\Phi}_1)_k^A T_i^k$. Since all of the VEVs are diagonal, and preserve $SU(2,3)$, we can separate $F_S = 0$ into two distinct constraint equations. Until now we have neglected to include coefficients in front of each of the terms in the lagrangian but in general they should be arbitrary, so the resulting constraint equations will have the form

$$Av_1 \tilde{v}_1 + B(v_1 \tilde{v}_1)^2 + C(v_1 \tilde{v}_1)(v_2 \tilde{v}_2) + Dv_T + Ev_T^2 + Fv_T v_1 \tilde{v}_1 + \dots = 0$$

and

$$Gv_2 \tilde{v}_2 + H(v_2 \tilde{v}_2)^2 + I(v_1 \tilde{v}_1)(v_2 \tilde{v}_2) + Jv_T + Kv_T^2 + Lv_T v_2 \tilde{v}_2 + \dots = 0$$

Notice that had we not chosen the VEV of T to be diagonal, $F_S = 0$ would enforce this to be true, as there are no other terms present in the off diagonal component equations. Also note that had we not chosen the diagonal components of the VEV to be equal, we would not have been able to split the constraints into two blocks as we have done above. Instead, we would have five independent equations, the equations within each block differing from each other only by the components replacing v_T . This would force these components to be equal.

The $F_T = 0$ equation is satisfied automatically by our choice of VEVs. Parallel to F_S , the only terms in the F_T equation come from W_T and W_{ST} . The terms from the latter are all zero due to the zero VEV of S , while the terms from the former must contain an even number of Φ fields but an odd number Φ_2 or $\tilde{\Phi}_2$ s and so will inevitably involve a product of Φ_1 or $\tilde{\Phi}_1$ times Φ_2 or $\tilde{\Phi}_2$.

Each of the F_Φ equations introduces a new constraint. The zero VEV of S will eliminate all terms in the F -equations from W_S and W_{ST} . The terms from W_{det} will vanish because each term will involve a product with at least one zero. This leaves W_Φ and W_T ; since these contain similar terms for each of the Φ s, we will focus in particular on F_{Φ_1} for illustration:

$$\begin{aligned}
 F_{\Phi_1} = & M\tilde{\Phi}_2 + \tilde{\Phi}_2 T + \frac{1}{M}[\tilde{\Phi}_2\Phi_2\tilde{\Phi}_2 + \tilde{\Phi}_2(\Phi_1\tilde{\Phi}_1) + \tilde{\Phi}_2(\Phi_2\tilde{\Phi}_2)] \\
 & + \frac{1}{M^2}[\tilde{\Phi}_2\Phi_2\tilde{\Phi}_2 T + \tilde{\Phi}_2 T(\Phi_1\tilde{\Phi}_1) + \tilde{\Phi}_2 T(\Phi_2\tilde{\Phi}_2) + \tilde{\Phi}_2(T\Phi_1\tilde{\Phi}_1) + \tilde{\Phi}_2(T\Phi_2\tilde{\Phi}_2)] \\
 & + \text{higher order.}
 \end{aligned}$$

Notice that the only surviving terms are proportional to some power of the VEV of $\tilde{\Phi}_2$ (this is true to all orders). Thus the bottom two diagonal components are identical to each other while the rest of the components are zero, the result of which is a single equation imposing some new constraint on the vs , of the form

$$0 = A\tilde{v}_2 + Bv_2\tilde{v}_2^2 + C\tilde{v}_2v_1\tilde{v}_1 + Dv_T\tilde{v}_2 + Ev_Tv_2\tilde{v}_2^2 + Fv_T\tilde{v}_2v_1\tilde{v}_1 + \dots$$

In addition to the two constraints we have from $F_S = 0$, here is a third equation involving all five degrees of freedom that must be satisfied in order for our chosen VEVs to lie at a minimum of the potential. The equations $F_{\tilde{\Phi}_1} = 0, F_{\Phi_2} = 0$ and $F_{\tilde{\Phi}_2} = 0$ each impose an additional new constraint, all having a similar form to that written above for Φ_1 . As expected, we end up with six constraints for five unknowns. This is always the case for a vacuum which preserves both SUSY and an R symmetry. Since our VEVs were designed to preserve an R symmetry, all six constraints are satisfied.

Let us now examine the mass spectrum of the low energy theory. Spontaneous breaking of the symmetry allows us to re-express the lagrangian by expanding the fields about the minimum of the potential, that is $\Phi \rightarrow \langle \Phi \rangle + \phi$, etc. The masses of the low energy fields are found by examining the coefficients of the terms quadratic in the fields, but we are not concerned with the specific value of the masses, as all masses will be of the order of the GUT scale and so will be integrated out.

Let us instead simply consider the various components that survive at low energies after inserting the VEVs. It is convenient to write the components of the fields appearing at low energies in terms of the Gell-Mann basis for the adjoint representation: $S_j^i = S^a(\lambda_j^i)_a, a = 1, 2 \dots 24, (25)$, where the λ_j^i are the 25 $U(5)$ generators. The allowed couplings can then be computed by tracing over the matrices and using the orthogonality conditions. Rather than examine every term in the superpotential individually, we highlight a few of the most important consequences. First, any term quadratic in the GUT scale fields automatically allows couplings for every component field, since $\text{Tr}\lambda^a\lambda^b = 2\delta^{ab}$ and so for example $(\Phi_1\tilde{\Phi}_2) = \Phi_1^a\tilde{\Phi}_2^b\text{Tr}(\lambda^a\lambda^b) \sim \Phi_1^a\tilde{\Phi}_2^a$. Second, any term with a trace containing the VEV of Φ_1 or $\tilde{\Phi}_1$ will prevent a coupling amongst the $SU(2)$ components; similarly a trace containing the VEV of Φ_2 or $\tilde{\Phi}_2$ will prevent $SU(3)$ couplings. This is due to the zero blocks of these VEVs. Finally, any term containing a trace over both the VEVs of Φ_1 or $\tilde{\Phi}_1$ and Φ_2 or $\tilde{\Phi}_2$, or a trace including the VEV of S , will be zero. Every mass term will

Configuration 1		Configuration 2		Configuration 3	
$SU_1(5)$	$SU_2(5)$	$SU_1(5)$	$SU_2(5)$	$SU_1(5)$	$SU_2(5)$
$H_u = 5$	$H_d = \bar{5}$	$H_u = 5$	$H_d = \bar{5}$	$H_u = 5$	$H_d = \bar{5}$
$\mathcal{U}_1 = 10$		$\mathcal{U}_1 = 10$			$\mathcal{U}_1 = 10$
$\mathcal{D}_1 = \bar{5}$		$\mathcal{D}_1 = \bar{5}$			$\mathcal{D}_1 = \bar{5}$
$\mathcal{D}_2 = \bar{5}$	$\mathcal{U}_2 = 10$	$\mathcal{D}_2 = \bar{5}$	$\mathcal{U}_2 = 10$	$\mathcal{D}_2 = \bar{5}$	$\mathcal{U}_2 = 10$
$\mathcal{U}_3 = 10$			$\mathcal{U}_3 = 10$		$\mathcal{U}_3 = 10$
$\mathcal{D}_3 = \bar{5}$			$\mathcal{D}_3 = \bar{5}$		$\mathcal{D}_3 = \bar{5}$

Table 3: Anomaly free matter configurations.

include either one field of Z'_4 R-charge 2 and one of R-charge 0 or two fields of R-charge 3 or 1 as discussed previously. We will not write out the results in detail, but they have been confirmed by explicit computation of the traces and diagonalization of the resulting mass matrix blocks. The low energy spectrum of our model indeed coincides with that of the Pentagon.

4. Matter fields

The purpose of this section is to consider the constraints on the low energy quark and lepton mass matrices. We have to embed the standard model fields and the penta-quarks in our model, without introducing anomalies in either the gauge symmetries or the discrete R symmetry.

4.1 R symmetry constraints

In the standard $SU(5)$ GUT theories the chiral matter consists of three ‘up’ fields $\mathcal{U}_i, i = 1, 2, 3$ that transform as 10s, three ‘down’ fields $\mathcal{D}_i, i = 1, 2, 3$ transforming as $\bar{5}$ s, and a pair of Higgs with $H_u \in 5$ and $H_d \in \bar{5}$. In our model the content will be the same, but we have some freedom to choose which of the two $SU(5)$ s to place these fields in. In order to cancel chiral anomalies there must be one $\bar{5}$ for each 5 or 10 in each $SU(5)$; this allows three possible configurations (table 3).

We want to ensure that at least the top quark mass is unsuppressed at low energies, so we will ignore the third of these possibilities since it does not allow a renormalizable Yukawa coupling for any of the up quarks. In fact, the only cases that will be of interest are models which allow $H_u \mathcal{U}_1 \mathcal{U}_1$ or $H_u \mathcal{U}_1 \mathcal{U}_3$ with three generations in $SU_1(5)$ (first configuration) or $H_u \mathcal{U}_1 \mathcal{U}_1$ with the generations mixed between the two $SU(5)$ ’s (second configuration). This requirement provides the first constraint on the R-charges of the matter fields, $H_u + \mathcal{U}_1 + \mathcal{U}_{1,3} = 2$.

The R-charges of the rest of the fields can be chosen by considering the desired low-energy mass matrices for the quarks and leptons. Depending on the choice of matter configuration between the two $SU(5)$ s, a low energy Yukawa coupling will exist only for a given combination of high energy fields, and this combination must sum to R-charge 2

for it to appear in the superpotential. In particular these terms will contain some number of Φ or $\tilde{\Phi}$ fields to mediate the interaction between matter fields in separate $SU(5)$ s. The number of such fields is determined by gauge invariance, but since $\Phi_1, \tilde{\Phi}_1$ and $\Phi_2, \tilde{\Phi}_2$ have different VEVs the choice between which to include when constructing the high energy terms will be determined by the desired low energy content (see tables 4, 5).

For instance, the Yukawa coupling $h_u Q_2 \bar{U}_2$ would be generated by a high energy term containing $H_u, \mathcal{U}_2, \mathcal{U}_2$, and the only gauge invariant construction (that is non-zero after inserting the Φ VEVs) is of the form $H_u \tilde{\Phi}_i \mathcal{U}_2 \mathcal{U}_2 \sim (\text{tr}(5 \times \bar{5}), \det[5 \times 5 \times 5 \times 5 \times \bar{5}])$. In this case the choice of i is clear, $i = 1$ would produce a coupling with the Higgs triplet while $i = 2$ would give the desired coupling to the Higgs doublet. Nevertheless, this choice has an important consequence: because $\tilde{\Phi}_2$ has R-charge 2, the sum of the matter field R charges must be zero, not two.

In some cases the choice is not so clear, and in fact the difference in R-charge between the Φ s can lead to mutually exclusive low energy couplings. Consider the coupling $H_u \mathcal{U}_1 \Phi_i \mathcal{U}_2 \Phi_j \sim (\det[5 \times 5 \times 5 \times 5 \times 5], \text{tr}[5 \times 5 \times 5 \times 5 \times \bar{5}])$. Evidently the choice $i = j$ would require the sum of R-charges $H_u + \mathcal{U}_1 + \mathcal{U}_2 = 2$, while $i \neq j$ requires $H_u + \mathcal{U}_1 + \mathcal{U}_2 = 0$. Obviously these conditions cannot both be satisfied, but the former (with $i = 1$) generates the low energy Yukawa coupling $h_u Q_1 \bar{U}_2$ while the latter leads to $h_u \bar{U}_1 Q_2$. To see this, let us represent the \mathcal{U}_i by the 5×5 matrix constructed of its low energy components,

$$\mathcal{U} = \left(\begin{array}{c|c} \bar{U} & Q \\ \hline Q & \bar{E} \end{array} \right)$$

In $SU_2(5)$ we will be multiplying $\langle \Phi_i \rangle \mathcal{U}_2 \langle \Phi_j \rangle$, so $i = j = 1$ selects out \bar{U}_2 , $i = j = 2$ selects \bar{E}_2 , and $i \neq j$ gives Q_2 . In $SU_1(5)$ we are taking a determinant of five vectors each with five components. Let us think of these as column vectors each with five rows. We know that the only non-zero contributions to a determinant will involve the multiplication of components from unique rows for each vector, i.e. there must be a component contribution from each row 1-5. Now the $\langle \Phi_1 \rangle$ only have non-zero components in rows 1-3 while the $\langle \Phi_2 \rangle$ will only contribute non-zero components from rows 4 and 5. However, to end up with the Higgs doublet the vector corresponding to H_u must contribute a component from either row 4 or 5 as well; thus the determinant including $i = j = 2$ will automatically be zero. If on the other hand $i = j = 1$, these will both contribute components from rows 1-3, so the vectors corresponding to \mathcal{U}_1 must have one contribution from rows 1-3 and one from rows 4-5, i.e. the components of Q_1 . If $i \neq j$, both contributions from \mathcal{U}_1 must be in rows 1-3, these components correspond to \bar{U}_1 .

In tables 4, 5 we have listed the high energy term responsible for each low energy Yukawa coupling as well as the necessary R-charge sum for the matter fields involved, dependent on the placement of matter in the two $SU(5)$ s.

Another constraint on the R-charges is that the 3 low energy operators $h_u L_{1,2,3}$ are forbidden. These would give a GUT scale mass to one of the Higgs fields and a matter

High Energy Term	R-charge Requirement	Low Energy Yukawa Couplings
$H_u \mathcal{U}_{1,3} \mathcal{U}_{1,3}$	$H_u + \mathcal{U}_{1,3} + \mathcal{U}_{1,3} = 2$	$h_u Q_{1,3} \bar{U}_{1,3}$
$H_u \mathcal{U}_{1,3} \Phi_1 \mathcal{U}_2 \Phi_1$	$H_u + \mathcal{U}_{1,3} + \mathcal{U}_2 = 2$	$h_u Q_{1,3} \bar{U}_2$
$H_u \mathcal{U}_{1,3} \Phi_1 \mathcal{U}_2 \Phi_2$	$H_u + \mathcal{U}_{1,3} + \mathcal{U}_2 = 0$	$h_u Q_2 \bar{U}_{1,3}$
$H_u \tilde{\Phi}_2 \mathcal{U}_2 \mathcal{U}_2$	$H_u + 2\mathcal{U}_2 = 0$	$h_u Q_2 \bar{U}_2$
$\mathcal{D}_{1,2,3} \mathcal{U}_{1,3} \tilde{\Phi}_2 H_d$	$H_d + \mathcal{U}_{1,3} + \mathcal{D}_{1,2,3} = 0$	$h_d Q_{1,3} \bar{D}_{1,2,3}, h_d L_{1,2,3} \bar{E}_{1,3}$
$\mathcal{D}_{1,2,3} \Phi_1 \mathcal{U}_2 H_d$	$H_d + \mathcal{U}_2 + \mathcal{D}_{1,2,3} = 2$	$h_d Q_2 \bar{D}_{1,2,3}$
$\mathcal{D}_{1,2,3} \Phi_2 \mathcal{U}_2 H_d$	$H_d + \mathcal{U}_2 + \mathcal{D}_{1,2,3} = 0$	$h_d L_{1,2,3} \bar{E}_2$

Table 4: Yukawa term R-charge constraints (1st configuration).

High Energy Term	R-charge Requirement	Low Energy Yukawa Couplings
$H_u \mathcal{U}_1 \mathcal{U}_1$	$H_u + 2\mathcal{U}_1 = 2$	$h_u Q_1 \bar{U}_1$
$H_u \mathcal{U}_1 \Phi_1 \mathcal{U}_{2,3} \Phi_1$	$H_u + \mathcal{U}_1 + \mathcal{U}_{2,3} = 2$	$h_u Q_1 \bar{U}_{2,3}$
$H_u \mathcal{U}_1 \Phi_1 \mathcal{U}_{2,3} \Phi_2$	$H_u + \mathcal{U}_1 + \mathcal{U}_{2,3} = 0$	$h_u Q_{2,3} \bar{U}_1$
$H_u \tilde{\Phi}_2 \mathcal{U}_{2,3} \mathcal{U}_{2,3}$	$H_u + \mathcal{U}_{2,3} + \mathcal{U}_{2,3} = 0$	$h_u Q_{2,3} \bar{U}_{2,3}$
$\mathcal{D}_{1,2} \Phi_1 \mathcal{U}_{2,3} H_d$	$H_d + \mathcal{U}_{2,3} + \mathcal{D}_{1,2} = 2$	$h_d Q_{2,3} \bar{D}_{1,2}$
$\mathcal{D}_{1,2} \Phi_2 \mathcal{U}_{2,3} H_d$	$H_d + \mathcal{U}_{2,3} + \mathcal{D}_{1,2} = 0$	$h_d L_{1,2} \bar{E}_{2,3}$
$H_d \tilde{\Phi}_2 \mathcal{U}_1 \tilde{\Phi}_1 \mathcal{D}_3$	$H_d + \mathcal{U}_1 + \mathcal{D}_3 = 0$	$h_d Q_1 \bar{D}_3$
$H_d \tilde{\Phi}_2 \mathcal{U}_1 \tilde{\Phi}_2 \mathcal{D}_3$	$H_d + \mathcal{U}_1 + \mathcal{D}_3 = 2$	$h_d L_3 \bar{E}_1$
$\mathcal{D}_{1,2} \mathcal{U}_1 \Phi_2 H_d$	$H_d + \mathcal{U}_1 + \mathcal{D}_{1,2} = 0$	$h_d Q_1 \bar{D}_{1,2}, h_d L_{1,2} \bar{E}_1$

Table 5: Yukawa term R-charge constraints (2nd configuration).

field, eliminating them from the low-energy spectrum.

Finally, there is also a constraint the R-charges must satisfy in order to ensure that the discrete Z_4 R-symmetry is anomaly free. This gives the equations (mod 4):

$$\begin{aligned}
 SU_1(5) : 0 &= 10\lambda + 5(\Phi_1 + \tilde{\Phi}_1 + \Phi_2 + \tilde{\Phi}_2 - 4) + 10p(T - 1) + 10(S - 1) \\
 &\quad + 5(P + \tilde{P} - 2) + (H_u - 1) + \sum_i 3(\mathcal{U}_i - 1) + \sum_j (\mathcal{D}_j - 1) \\
 SU_2(5) : 0 &= 10\lambda + 5(\Phi_1 + \tilde{\Phi}_1 + \Phi_2 + \tilde{\Phi}_2 - 4) + 10(1 - p)(T - 1) \\
 &\quad + (H_d - 1) + \sum_i 3(\mathcal{U}_k - 1) + \sum_j (\mathcal{D}_l - 1).
 \end{aligned}$$

The λ s represent the gauginos, which must have R-charge 1 (the vector fields having R-charge 0) since they arise in the D-term of the superpotential. $p = 0, 1$ indicating which SU(5) the T transforms in (notice that we could have created a similar parameter for the S and P, \tilde{P} , but their contributions will sum to zero anyway). i, j run over the matter fields in $SU_1(5)$, and k, l $SU_2(5)$. Inserting the known R-charge values for the fields, these

simplify to:

$$\begin{aligned}
 SU_1(5) : 1 + 2p + H_u + \sum_i 3(\mathcal{U}_i - 1) + \sum_j (\mathcal{D}_j - 1) &= 0 \\
 SU_2(5) : 1 + 2(1 - p) + H_d + \sum_k 3(\mathcal{U}_k - 1) + \sum_l (\mathcal{D}_l - 1) &= 0
 \end{aligned}$$

These constraints allow us to determine the R-charge of two of the matter fields in terms of the others (recall that H_u can be determined by the requirement of a renormalizable top quark Yukawa term, and H_d can be determined by $\tilde{\Phi}_1 + H_u + H_d = 2$). On the other hand, we can use the last of these conditions to combine the two equations into a single constraint on the matter fields

$$2 = 3(\mathcal{U}_1 + \mathcal{U}_2 + \mathcal{U}_3) + (\mathcal{D}_1 + \mathcal{D}_2 + \mathcal{D}_3).$$

Hence, there is an anomaly-free model for any choice of R-charges which satisfy this condition on the matter fields. However, the R-charges of the Higgs fields are then uniquely determined from the matter fields.

At this point it should be clear that, although our model has the field content of the Pentagon model, the low energy R charge assignments must be quite different. In particular, we find that *the low energy R charges cannot be generation blind*. This is a consequence of the GUT structure, and the anomaly constraints. There is some freedom to shift the discrete anomaly constraints by assigning the axion superfield X an additive transformation law under Z_4 . However, as we will see in the next section, this cannot solve the most severe phenomenological problems of this model.

4.2 Phenomenology

An important phenomenological constraint on any GUT is that it satisfy the experimental bounds on proton decay. The lower bound on the overall lifetime of the proton is currently 2.1×10^{29} years [13]. However, there are stronger bounds for specific decays, the strongest of which is 1.6×10^{33} years for $p \rightarrow e^+ \pi$ [14]. The triplet Higgs is no danger to proton decay in this model since it naturally acquires a GUT-scale mass. However, depending on the choice of R-charges for the matter fields, there are a number of potentially dangerous baryon and lepton violating operators that could mediate proton decay. Dimension-6 operators are suppressed by $\frac{1}{M_U^2}$, so the decay rates are suppressed roughly by $(\frac{m_p}{M_U})^4 \approx 10^{-64}$.⁷ This is right in the neighborhood of the current bound, meaning that it is not ruled out yet but predicts that proton decay should be seen soon. However, dimension-4 and dimension-5 operators which violate baryon and lepton number should not be allowed as they involve fewer inverse-powers of M_U and would permit proton decay at a rate far outside of current bounds. An exception to this is dimension-5 *purely baryon* number violating operators, and dimension-5 *purely lepton* number violating operators. In these cases, two vertices each with an inverse-power of M_U are required in the same diagram, making the overall lifetime

⁷Recall that in the Pentagon model, the unified coupling is on the edge of the perturbative regime, so there is no significant coupling constant suppression of proton decay rates.

operator	violation	dimension
$LL\bar{E}$	L	4
$LQ\bar{D}$	L	4
$\bar{D}\bar{D}\bar{U}$	B	4
$\bar{U}\bar{E}D$	BL	5
$LQQQ$	BL	5
$\bar{E}\bar{U}\bar{U}\bar{D}$	BL	5
not dangerous:		
$Q\bar{U}\bar{L}$	L	5
h_dQQQ	B	5
$h_u h_u LL$	L	5

Table 6: B and L violating operators.

on the same order as what a dimension-6 operator which violates *both* baryon and lepton number would yield. Table 6 enumerates the dangerous dimension 4 and 5 operators that could appear in the theory.

We have determined that it is not possible to forbid all of the dangerous operators with any combination of Z_4 R-charges, even ignoring the discrete anomaly constraints. Most of these can be forbidden if we impose matter parity — a discrete (non-R) Z_2 symmetry where all of the matter fields have charge 1 and all other fields remain uncharged. Then, all but the $UUUD$ operators are eliminated, while not forbidding any of the Higgs Yukawa couplings ($H_u\mathcal{UU}$ or $H_d\mathcal{UD}$). This leaves less work for the R symmetry to do.⁸ After adding the Z_2 , it is then possible to forbid all the remaining dangerous operators with the Z_4 , however all of those models have a Z_4 anomaly (and the Z_2 is anomalous as well). Later, we will discuss possible ways of fixing these anomaly problems. Another possibility is to add a discrete symmetry (either instead of the Z_2 or in addition to it) which gets rid of $UUUD$ but forbids some of the Higgs Yukawa couplings, which may or may not already be forbidden by the R-symmetry.

We do want to allow the seesaw operators $h_u h_u LL$. These give a tiny Majorana mass matrix for the neutrinos. Therefore, we wish to allow this operator for as many combinations of L generations as possible.

Another phenomenological issue is that of neutron-anti-neutron oscillations. This is similar to proton decay in that dimension-6 operators are okay whereas dimension-4 and dimension-5 operators are not. Only baryon number violation is relevant for neutron oscillations, however $n \rightarrow \bar{n}$ requires the baryon number change by 2. So the dimension-5 baryon number violating operator h_dQQQ listed as “not dangerous” for proton decay is

⁸The Z_2 matter parity is optimal in the sense that expanding it to any larger symmetry cannot help forbid the $UUUD$ operators without also forbidding either some of the Higgs Yukawa couplings or $\tilde{\Phi}_1 H_u H_d$. The reason for this is that $\tilde{\Phi}_1$ cannot get a charge under this new symmetry without forbidding some of the low energy Yukawa couplings or allowing some of the B or L violating operators by giving T a charge. And without a charge for Φ_1 , the condition that $\Phi_1 H_u H_d$, $H_u\mathcal{UU}$, and $H_d\mathcal{UD}$ be chargeless forces $UUUD$ to be chargeless.

also safe for neutron oscillations because it violates baryon number only by 1. Hence, nothing new is added by this constraint.

Next we examine the constraints on quark and lepton mass matrices. The generation dependence of R charges implies that many of the entries in these mass matrices are zero. Which ones are non-zero, how many non-zero mass eigenvalues, and the approximate ratios of mass eigenvalues depends on the choice of R-charges for the matter fields. We have used a brute-force computer algorithm to explore the various possibilities. The first limitation we have found is that, even ignoring B or L violation as well as the discrete anomaly constraints, it is impossible to get rank 3 mass matrices for both the up quarks and the down quarks at the same time. The requirement that the SU(5) anomalies cancel implies that there can be at most 5 massive quarks. Models with 5 massive quarks limit the number of massive neutrinos to 2. If we choose the up quark to be massless, then the number of massive leptons is also limited to 2. If instead we choose a massless down quark, then all 3 leptons can be massive. Note that in this analysis we have not imposed the constraint that the B and L violating operators be forbidden — when these constraints are combined, the restrictions are much more severe.

If we impose the anomaly constraints, there are 512 different models with an unsuppressed top quark mass in the first configuration of table 3 and 896 models in the second configuration. We have found candidates where a discrete symmetry other than matter parity is used to forbid some of the dangerous operators. One possibility is to use a Z'_2 symmetry where all of the \mathcal{U} fields are odd, and all the rest of the fields are even. This forbids all of the $\mathcal{H}_d \mathcal{U} \mathcal{D}$ Yukawa couplings, but still allows $H_u \mathcal{U} \mathcal{U}$. It forbids all of the dangerous operators except $\mathcal{U} \mathcal{U} \mathcal{D}$, which can either be forbidden by the other Z_2 or by the Z_4 R-symmetry. There are a number of models where it is forbidden by the Z_4 alone, 16 of which look potentially interesting (see appendix, table 10). 8 of these involve just a mass for the top quark, all the other quarks and leptons massless, and either 2 or 3 neutrinos. The other 8 involve both top and charm masses, the other quarks and leptons massless, and 2 or 3 neutrinos. In half of those 8, the charm quark is suppressed by a factor of ϵ^2 relative to the top quark, which appears preferable to the other half in which it's suppressed by just a single power of ϵ . If instead of just relying on Z'_2 (and the Z_4 R symmetry) both Z_2 and the Z'_2 are imposed, there are no restrictions on the R charges and there are many more possibilities for the up-quark, lepton, and neutrino masses. The main limitation with any models involving the Z'_2 is that all 3 generations of down-type quarks and leptons must remain massless.

There are other candidate models which involve adding discrete horizontal symmetries (see appendix, table 13). In contrast to adding discrete symmetries that act the same way on each generation, it takes more creativity to find horizontal symmetries that work phenomenologically and we have not yet found a way of automating the process. Our approach so far has been to look at models with only 1 $\mathcal{U} \mathcal{U} \mathcal{U} \mathcal{D}$ operator (since all the rest can be forbidden by the Z_2), and to use a horizontal symmetry to eliminate this operator. The symmetry will almost always remove some Yukawa couplings, but for the case of only a single $\mathcal{U} \mathcal{U} \mathcal{U} \mathcal{D}$ operator, it can be chosen carefully so as not to reduce the rank of the mass matrix. An explicit example which employs a Z_{3H} horizontal symmetry is shown in

field	R	Z_2	Z_{3H}
H_u	2	0	0
H_d	0	0	0
\mathcal{U}_1	0	1	0
\mathcal{U}_2	0	1	0
\mathcal{U}_3	0	1	1
\mathcal{D}_1	1	1	0
\mathcal{D}_2	1	1	0
\mathcal{D}_3	0	1	0

Table 7: Example of a model with Z_{3H} horizontal symmetry (configuration 2) in addition to matter parity. Model contains a heavy top, a lighter bottom and tau, two neutrinos, and the other two generations of quarks and leptons massless.

table 7. The Z_{3H} symmetry disallows the \mathcal{U}_3 field from coupling to any other matter fields except as a cubic. This has the effect of eliminating the remaining dangerous operator, but also removes the mass for the charm quark (which in this particular case is good because it would have been on the order of the top mass). Furthermore, unlike the Z_2 and Z'_2 , this Z_{3H} symmetry is itself anomaly free (both the Z_2 and Z'_2 have an odd number of charged fields in each SU(5), whereas the only field charged under the Z_{3H} in this model is a \mathcal{U} which contains a factor of three in the anomaly equation). The end result is a model with a heavy top quark, a bottom and tau quark with suppressed masses, and 2 neutrino masses. Without violating anomalies or adding more fields, models of this type are the closest to the real world that we have been able to find. A slight variation on this model with the same R-charges is to use a Z_{4H} horizontal symmetry instead of the Z_{3H} (see appendix, table 12). This results in a model with a heavy top quark, a bottom and tau quark suppressed by ϵ , and a charm quark suppressed by ϵ^2 . However, the Z_{4H} symmetry is itself anomalous.

We have found that shifting the discrete anomaly equations by giving the X field a transformation law under the Z_4 , does not help us to obtain full rank mass matrices. It also does not help fix the anomaly in the Z_2 or Z_{4H} symmetries, although it can be used to fix the Z'_2 anomaly. If these models are to be made realistic, one would have to imagine that the missing matrix elements of the quark and lepton mass matrices came from breaking of the R symmetry. In CSB explicit R symmetry breaking is expected to vanish like a power of the cosmological constant. The gravitino mass is $\sim \Lambda^{1/4}$. In the dimensionless quark and lepton Yukawa couplings we might imagine $\Lambda^{1/4}/m$. Even if we take m of order a TeV, this is too small to be of phenomenological help. We would have to postulate corrections that scale with even smaller powers of Λ . Furthermore, since we have used the R symmetry to eliminate dangerous B and L violating operators, one would have to explain why these R violating corrections did not lead to rapid proton decay.

5. Adding a 10 and a $\overline{10}$

An interesting possibility for resolving the issue of full rank mass matrices is to add an

extra pair of Higgs fields. As discussed in the previous section, because the $SU(5)$ anomalies force us to place the matter fields in different gauge groups (and in particular forces us to split one of the generations), certain Yukawa terms are formed only by inserting VEVs of the Φ fields to bridge the two $SU(5)$ s, and this prevents certain low energy couplings due to R-charge. However, by inserting an extra pair of $\bar{5}, 5$ fields into the two $SU(5)$ s, we can place all of the matter in a single $SU(5)$. As before we will want to ensure that all Higgs triplets gain a GUT scale mass, so we will enforce that $H_u + H_d = 2$, as well as $G_d + G_u = 2$ where G_d is a $\bar{5}$ in $SU_1(5)$ and G_u a 5 in $SU_2(5)$. We will also impose that $H_u + G_d \neq 2$ and $H_d + G_u \neq 2$ since these could give mass to the doublets. Then, rewriting the Z_4 anomaly constraints,

$$\begin{aligned} SU_1(5) : 0 &= 2p + H_u + G_d + 3(\mathcal{U}_1 + \mathcal{U}_2 + \mathcal{U}_3) + (\mathcal{D}_1 + \mathcal{D}_2 + \mathcal{D}_3) \\ SU_2(5) : 0 &= 2(1 - p) + H_d + G_u \end{aligned}$$

and imposing the conditions asserted above, we find that $p = 1$, $H_d = -G_u$, $H_u = -G_d$, and so $2 = 3(\mathcal{U}_1 + \mathcal{U}_2 + \mathcal{U}_3) + (\mathcal{D}_1 + \mathcal{D}_2 + \mathcal{D}_3)$. That is, the R-charges of the Higgs fields are determined in terms of one another, but are completely independent of the charges of the matter fields.

There is, unfortunately, a significant phenomenological problem with this idea: in the model above all of the triplet Higgs do gain mass, but this means all of the doublets remain massless. Introducing a new pair of low energy Higgs doublets introduces eight new degrees of freedom, four of which are new charged bosons, which would probably be a disaster in terms of flavor changing neutral currents. In fact this problem is unavoidable, even if we relax the anomaly constraints imposed above. In order to ensure that all triplet Higgs gain mass, we must have $H_u + H_d = 2$ and $G_d + G_u = 2$, or $H_u + H_d + G_d + G_u = 0$. This implies that at low energies $h_u + h_d = -g_u - g_d$. However, we want our original pair of Higgs doublets to have R-charge such that $h_u + h_d = 0$ to ensure that they remain massless at the TeV scale as well as to allow a μ term $sh_u h_d$; therefore *both* Higgs doublets must remain massless as long as we insist on avoiding light triplets. Possible resolutions of this issue are discussed in [12].

This does lead us to a slightly different approach, however. Instead of an extra pair of Higgs fields, let us introduce two fields $A \in (\bar{10}, 1)$ and $B \in (1, 10)$ such that $A + B = 2$. These fields will have zero VEVs so will not affect our SUSY vacuum, and they will gain GUT scale mass because both the terms $A\Phi_1\Phi_1B$ and $A\Phi_2\Phi_2B$ are allowed (actually this is a total of four terms since there are two different ways to contract the indices). The important point is that, as in the model with two pairs of Higgs doublets, all of the matter fields can be placed in a single $SU(5)$. The Z_4 anomaly constraints are now

$$\begin{aligned} SU_1(5) : 0 &= 2 + 2p + (H_u - 1) + 3(A - 1) + 3(\mathcal{U}_1 + \mathcal{U}_2 + \mathcal{U}_3 - 3) + (\mathcal{D}_1 + \mathcal{D}_2 + \mathcal{D}_3 - 3) \\ SU_2(5) : 0 &= 2 + 2(1 - p) + (H_d - 1) + 3(B - 1) \end{aligned}$$

The second of these defines a relation between the Higgs and the $10, \bar{10}$ fields. Combining this with the requirements that $H_u + H_d = 2$ and $A + B = 2$, we now find that H_u and A

Configuration 1		Configuration 2		Configuration 3		Configuration 4	
$SU_1(5)$	$SU_2(5)$	$SU_1(5)$	$SU_2(5)$	$SU_1(5)$	$SU_2(5)$	$SU_1(5)$	$SU_2(5)$
$H_u = 5$	$H_d = \bar{5}$	$H_u = 5$	$H_d = \bar{5}$	$H_u = 5$	$H_d = \bar{5}$	$H_u = 5$	$H_d = \bar{5}$
$A = \bar{10}$	$B = 10$	$A = \bar{10}$	$B = 10$	$A = \bar{10}$	$B = 10$	$A = \bar{10}$	$B = 10$
$\mathcal{U}_1 = 10$		$\mathcal{U}_1 = 10$			$\mathcal{U}_1 = 10$		$\mathcal{U}_1 = 10$
$\mathcal{D}_1 = \bar{5}$		$\mathcal{D}_1 = \bar{5}$			$\mathcal{D}_1 = \bar{5}$		$\mathcal{D}_1 = \bar{5}$
$\mathcal{U}_2 = 10$		$\mathcal{U}_2 = 10$			$\mathcal{U}_2 = 10$		$\mathcal{U}_2 = 10$
$\mathcal{D}_2 = \bar{5}$		$\mathcal{D}_2 = \bar{5}$			$\mathcal{D}_2 = \bar{5}$		$\mathcal{D}_2 = \bar{5}$
$\mathcal{U}_3 = 10$			$\mathcal{U}_3 = 10$	$\mathcal{U}_3 = 10$			$\mathcal{U}_3 = 10$
$\mathcal{D}_3 = \bar{5}$			$\mathcal{D}_3 = \bar{5}$	$\mathcal{D}_3 = \bar{5}$			$\mathcal{D}_3 = \bar{5}$

Table 8: Anomaly free matter configurations with a 10 and $\bar{10}$.

cancel each other in the first equation, and so we are left with

$$A = H_u + 2p \tag{5.1}$$

$$2 = 3(\mathcal{U}_1 + \mathcal{U}_2 + \mathcal{U}_3) + (\mathcal{D}_1 + \mathcal{D}_2 + \mathcal{D}_3) \tag{5.2}$$

just as for the case of two Higgs doublet pairs above. Again, the R-charges of the H_u, H_d, A, B are all related but independent of the charges of the matter fields. Remarkably, the relationship between the R-charges of the matter fields turns out to be the same regardless of whether we add the extra pair of Higgs, the A and B fields, or neither. The results will be discussed below.

First let us note that while this discussion has so far been specific to models with all matter in a single $SU(5)$, the inclusion of A and B actually provides more freedom for the placement of the fields. In fact, all that is required by gauge anomaly cancellation is that each *generation* of matter be placed in a single $SU(5)$ (table 8). We will ignore configuration 4 because the top quark mass is suppressed. Configuration 1 has been discussed so far. More generally, for any of the configurations, the anomaly equations can be put in the form:

$$\begin{aligned} SU_1(5) : 2 + 2p + H_u + 3A + \sum_i 3(\mathcal{U}_i - 1) + \sum_j (\mathcal{D}_j - 1) &= 0 \\ SU_2(5) : 2 + 2(1 - p) + H_d + 3B + \sum_k 3(\mathcal{U}_k - 1) + \sum_l (\mathcal{D}_l - 1) &= 0 \end{aligned}$$

These equations can be combined to give exactly the same relation between the matter fields as above (5.2). In our models without A and B, the Higgs R-charges were then determined in terms of the matter fields. However, in the present case, the Higgs R-charges can also be freely chosen (as long as they add up to 2) and it's A and B which are uniquely determined. This would appear to give rise to four times as many anomaly-free distinguishable low energy models. This is exciting, however in actuality it only adds twice as many new models, because of some redundancy due to the p parameter. Another advantage is that, in all of these configurations, our Z_2 matter parity is anomaly free (due to there being an even number of fields in each $SU(5)$).

Our most significant result is the discovery of models which are free of all B and L violation and have entirely full mass matrices. Unfortunately, these models suffer from

an anomaly in the R-symmetry, but we believe this can be remedied with the axion. Without the axion, we find that it is still not possible to form Z_4 anomaly-free models with full rank mass matrices, regardless of the matter configuration. It is possible to form anomaly-free models with everything massive except the up quark. However, even these models are problematic, as they all allow \mathcal{UUUD} B and L violating operators; as discussed previously, these operators cannot be eliminated with generation blind discrete symmetries. Nevertheless, these models were not even possible before the inclusion of the A and B fields. The bottom line is that, by allowing each matter generation to sit in a single $SU(5)$, we have introduced a plethora of new models, both with and without anomalies, that appear much more realistic than what was previously possible. The results of our computer search can be summarized as follows:

- There is a class of anomaly-free models in configurations 1,3 that have masses for 2 up quarks, 3 down quarks, 3 leptons, and 2 or 3 neutrinos; however, there are \mathcal{UUUD} B and L violating operators that need to be removed with a horizontal symmetry (see appendix, table 17).
- There are Z_4 anomaly-free models from all three configurations that have non-trivial mass matrices and no \mathcal{UUUD} operators. The remaining B and L violating operators can be forbidden by our (anomaly-free) Z_2 (see appendix, table 18).
- All three configurations have anomaly-free models without dangerous baryon violating operators, and only 3 (configuration 1) or 9 (configurations 2,3) lepton violating operators. Most of these have all quarks and leptons massless, but there are a few in configurations 2,3 with 2 up quarks, 1 down quark, 0 leptons, and 2 neutrinos. See appendix, table 19.
- There are no anomaly-free models that are also free of lepton violation, but a large class of them (with and without baryon violation) if we ignore the anomaly constraint. These might be useful for resolving puzzles associated with the current experimental bound on the Higgs mass.
- There are very interesting models in configuration 1 (all matter in $SU_1(5)$) that have a Z_4 anomaly but there are NO baryon and lepton violating operators and ALL Yukawa couplings (every entry in each matrix) are non-zero (table 9). These models have a possible Froggatt-Nielsen mechanism. Certain Yukawa couplings involve one higher power of the GUT scale fields Φ , so if the VEV is ϵ in units of M_U , some matrix elements will be suppressed. One finds no suppression for up quarks or for the neutrino seesaw terms and one power of ϵ for downquarks and leptons. This could supply part of the explanation of the texture of mass matrices. For the rest we would have to invent more Froggatt-Nielsen symmetries.

The models of table 9 can be rendered anomaly free by assigning a Z_4 transformation law to the axion. In these models all matter fields have identical R-charge (= 1 or 3), and are precisely the charge assignments in the original Pentagon papers. Specifically, for all

config.	H_u	H_d	p	A	B	\mathcal{U}_1	\mathcal{U}_2	\mathcal{U}_3	\mathcal{D}_1	\mathcal{D}_2	\mathcal{D}_3	B	L	Ups	Downs	Lept.	Neut.
1st	0	2	0	0	2	1	1	1	1	1	1	0	0	3	3	3	3
1st	0	2	0	0	2	3	3	3	3	3	3	0	0	3	3	3	3
1st	0	2	1	2	0	1	1	1	1	1	1	0	0	3	3	3	3
1st	0	2	1	2	0	3	3	3	3	3	3	0	0	3	3	3	3

Table 9: Ideal models: no B or L violation and full quark and lepton mass matrices. All the up quarks are unsuppressed, downquarks and leptons are suppressed by ϵ , and neutrinos are unsuppressed. These models are ruled out by the instanton anomaly but are made possible by adding an axion.

matter fields to have the same R-charge we want $3(\mathcal{U}_1 + \mathcal{U}_2 + \mathcal{U}_3) + (\mathcal{D}_1 + \mathcal{D}_2 + \mathcal{D}_3) = 0$, instead of 2. This can be arranged with an axion shift of π in units of f_a in $SU_1(5)$. These models are ideal in a number of ways and stand out as clearly superior to all other models discussed in this paper.

Since the axion decay constant is large, one might have worried that this mechanism will lead to large spontaneous breaking of the Z_4 symmetry. In other words, we can replace non-invariant operators by invariant ones, simply by multiplying with the appropriate power of e^{X/f_a} . This is mathematically correct, however, if X is really to serve as a QCD axion, no such terms can appear in the effective action above the QCD scale. If they did, they would provide a potential for the axion which dominates that generated by QCD, and X would not solve the strong CP problem. In our model, we include X in the GUT scale Lagrangian only via a term

$$\int d^2\theta (X/f_a) W_\alpha^2 + h.c.$$

involving the gauge field strength of $SU_1(5)$. The classical Lagrangian has a $U(1)$ shift symmetry, which is preserved to all orders in perturbation theory, and broken predominantly by QCD⁹. One has to appeal to a more fundamental UV complete model, to justify the argument that there are no other couplings of X allowed in the GUT scale effective Lagrangian.

6. Conclusions

In this paper, we have examined a large variety of SUSic grand unified models, which solve the doublet triplet splitting problem and reduce to the Pentagon model at low energies. Most of the models have phenomenological problems, and all of them have a large number of GUT scale fields. We are not particularly bothered by the latter problem because we view our models as a stepping stone to higher dimensional models originating in string theory.

The most successful class of models involved the addition of GUT scale fields A and B , transforming in the $(\overline{10}, 1) \oplus (1, 10)$ of $SU_1(5) \times SU_2(5)$. If we use the QCD axion to cancel

⁹Here we are assuming that SUSY is broken by the ISS mass term. In the supersymmetric limit of the Pentagon model, QCD is IR free and no potential is generated for X .

the discrete anomaly, we obtain models which preserve Baryon and Lepton number up to and including dimension 5 operators. They also have quark and lepton mass matrices of full rank. The models *could* predict a hierarchy between up quark masses and those of down quarks and charged leptons,¹⁰ if a certain VEV is small in GUT units. It is not clear whether it makes sense to attribute the entire ratio m_t/m_b to a single power of the VEV of a scalar field, and one would have to understand more about the microscopic origin of the model before claiming this as a victory.

The rest of the texture of the quark and lepton mass matrices might be explainable in terms of horizontal symmetries and the Froggatt-Nielsen mechanism. In models without the A and B fields, we have been forced to introduce discrete symmetries to eliminate dangerous B and L violating operators. For the most part, these led to unpleasant results for mass matrices, with too many massless particles. As far as we can see, the only viable strategy for such models is to postulate terms which explicitly break the Z_4 symmetry, above and beyond the ISS mass term. In order to get acceptable results, we would probably have to postulate R breaking terms that scale to zero even more slowly than the (already mysterious) $\Lambda^{1/4}$. Of course, if we abandon the origins of the Pentagon model in CSB, and attribute the R breaking to dynamical mechanisms in effective field theory, the range of possibilities is wider. We have not explored this option.

Our preference is to pursue the addition of horizontal symmetries to the models with A and B fields, which have full rank mass matrices in their current form. This will be the subject of future work. Another direction we want to pursue is a loosening of our requirements on dimension four B and L violation. This might be useful for resolving puzzles associated with the current experimental bound on the Higgs mass.

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A. Interesting models

In the appendix we list for reference a variety of classes of models that we find interesting. The list is not comprehensive, but contains models we believe may be useful in a future search for a Froggatt-Nielsen mechanism. For each model we list its SU(5) matter configuration (tables 3, 8 in the text), the R-charges of the relevant fields, the number of B (baryon) and L (lepton) violating operators allowed by the R-symmetry (before imposing other discrete symmetries), the number of $UUUD$ operators where relevant, and the ranks of the up quark, down quark, lepton, and neutrino mass matrices.

A.1 Models without a 10 and $\bar{10}$

¹⁰Recall that in the Pentagon model we expect that $\tan\beta \sim 1$, so that the up-down hierarchy must be attributed to Yukawa couplings.

config	H_u	H_d	p	\mathcal{U}_1	\mathcal{U}_2	\mathcal{U}_3	\mathcal{D}_1	\mathcal{D}_2	\mathcal{D}_3	B	L	Ups	Downs	Lept.	Neut.
1st	0	2	0	2	2	1	1	1	1	30	33	2	0	0	3
1st	2	0	1	2	2	1	1	1	1	30	33	1	0	0	3
1st	0	2	0	1	2	2	1	1	1	30	33	2	0	0	3
1st	2	0	1	1	2	2	1	1	1	30	33	1	0	0	3
1st	0	2	0	3	2	2	3	3	3	30	33	2	0	0	3
1st	2	0	1	3	2	2	3	3	3	30	33	1	0	0	3
1st	0	2	0	2	2	3	3	3	3	30	33	2	0	0	3
1st	2	0	1	2	2	3	3	3	3	30	33	1	0	0	3
2nd	2	0	1	0	0	0	1	3	2	18	27	1	0	0	2
2nd	2	0	1	0	0	0	3	1	2	18	27	1	0	0	2
2nd	2	0	1	2	2	2	1	1	2	1	7	1	0	0	2
2nd	2	0	1	2	2	2	3	3	2	1	7	1	0	0	2
2nd	2	0	1	2	2	1	1	1	1	32	47	2	0	0	3
2nd	2	0	1	2	1	2	1	1	1	32	47	2	0	0	3
2nd	2	0	1	2	3	2	3	3	3	32	47	2	0	0	3
2nd	2	0	1	2	2	3	3	3	3	32	47	2	0	0	3

Table 10: 16 models with masses for either a top and charm or just a top and no down-type quarks or lepton masses. The charm is suppressed by ϵ for models in configuration 1 and by ϵ^2 for models in configuration 2. All B,L violating operators are forbidden by imposing a Z'_2 with $\mathcal{U} = 1$, $\mathcal{D} = 0$, which also eliminates all $H_d \mathcal{U} \mathcal{D}$ (note: the number of B and L violating operators listed are before applying this symmetry).

field	R	Z_2	Z_{3H}
H_u	2	0	0
H_d	0	0	0
\mathcal{U}_1	0	1	0
\mathcal{U}_2	0	1	0
\mathcal{U}_3	0	1	1
\mathcal{D}_1	1	1	0
\mathcal{D}_2	1	1	0
\mathcal{D}_3	0	1	0

Table 11: An example of a model with a Z_{3H} horizontal symmetry (configuration 2) in addition to matter parity. Model contains 1 up quark, 1 down quark and lepton suppressed by ϵ , and two neutrinos. All B and L violating operators are forbidden.

field	R	Z_2	Z_{4H}
H_u	2	0	0
H_d	0	0	0
\mathcal{U}_1	0	1	2
\mathcal{U}_2	0	1	1
\mathcal{U}_3	0	1	3
\mathcal{D}_1	1	1	0
\mathcal{D}_2	1	1	0
\mathcal{D}_3	0	1	1

Table 12: An example of a model with Z_{4H} horizontal symmetry (configuration 2) in addition to matter parity. Model contains heavy top, bottom and tau masses suppressed by ϵ , charm mass suppressed by ϵ^2 , 2 neutrinos, and the rest of the quarks and leptons massless. All B and L violating operators are forbidden. The Z_4 R symmetry is anomaly free, however the Z_{4H} symmetry is anomalous.

config.	H_u	H_d	p	\mathcal{U}_1	\mathcal{U}_2	\mathcal{U}_3	\mathcal{D}_1	\mathcal{D}_2	\mathcal{D}_3	B	L	\mathcal{UUUD}	Ups	Downs	Lept.	Neut.
1st	2	0	0	0	0	0	1	1	0	16	28	1	2	1	1	2
1st	2	0	0	0	0	0	3	3	0	16	28	1	2	1	1	2
1st	2	0	0	0	0	0	0	1	1	16	28	1	2	1	1	2
1st	2	0	0	0	0	0	1	0	1	16	28	1	2	1	1	2
1st	2	0	0	0	0	0	0	3	3	16	28	1	2	1	1	2
1st	2	0	0	0	0	0	3	0	3	16	28	1	2	1	1	2
2nd	2	0	1	2	2	2	1	1	2	1	7	1	1	1	0	2
2nd	2	0	1	2	2	2	3	3	2	1	7	1	1	1	0	2

Table 13: Anomaly-free models that have only one \mathcal{UUUD} operator. These are good prospects for adding a discrete horizontal symmetry.

config	H_u	H_d	p	\mathcal{U}_1	\mathcal{U}_2	\mathcal{U}_3	\mathcal{D}_1	\mathcal{D}_2	\mathcal{D}_3	B	L	Ups	Downs	Lept.	Neut.
1st	0	2	0	2	2	2	1	1	1	0	0	1	0	0	3
1st	0	2	1	2	2	2	1	1	1	0	0	1	0	0	3
1st	0	2	0	2	2	2	3	3	3	0	0	1	0	0	3
1st	0	2	1	2	2	2	3	3	3	0	0	1	0	0	3

Table 14: 4 models with a massive top quark and 3 neutrinos which have no lepton or baryon-violating operators. However, the top quark mass is suppressed by ϵ . These models are anomalous without the axion shift.

config	H_u	H_d	p	\mathcal{U}_1	\mathcal{U}_2	\mathcal{U}_3	\mathcal{D}_1	\mathcal{D}_2	\mathcal{D}_3	B	L	Ups	Downs	Lept.	Neut.
2nd	2	0	0	2	2	2	3	3	1	12	0	1	0	0	3
2nd	2	0	1	2	2	2	3	3	1	12	0	1	0	0	3
2nd	2	0	0	2	2	2	1	1	3	12	0	1	0	0	3
2nd	2	0	1	2	2	2	1	1	3	12	0	1	0	0	3

Table 15: 4 models with a massive top quark and 3 neutrinos which have no lepton or baryon-violating operators. However, the top quark mass is suppressed by ϵ . These models are anomalous without the axion shift.

A.2 Models with a 10 and $\overline{10}$

config.	H_u	H_d	p	A	B	\mathcal{U}_1	\mathcal{U}_2	\mathcal{U}_3	\mathcal{D}_1	\mathcal{D}_2	\mathcal{D}_3	B	L	Ups	Downs	Lept.	Neut.
1st	0	2	0	0	2	1	1	1	1	1	1	0	0	3	3	3	3
1st	0	2	0	0	2	3	3	3	3	3	3	0	0	3	3	3	3
1st	0	2	1	2	0	1	1	1	1	1	1	0	0	3	3	3	3
1st	0	2	1	2	0	3	3	3	3	3	3	0	0	3	3	3	3

Table 16: Ideal models. No B or L violation and full quark and lepton mass matrices. All the up quarks are unsuppressed, downquarks and leptons are suppressed by ϵ , and neutrinos are unsuppressed. These models are ruled out by the instanton anomaly but are made possible by adding an axion.

config.	H_u	H_d	p	A	B	\mathcal{U}_1	\mathcal{U}_2	\mathcal{U}_3	\mathcal{D}_1	\mathcal{D}_2	\mathcal{D}_3	B	L	Ups	Downs	Lept.	Neut.
3rd	2	0	0	2	0	1	1	1	1	1	3	21	21	2	3	3	3
3rd	2	0	1	0	2	1	1	1	1	1	3	21	21	2	3	3	3
3rd	2	0	0	2	0	3	3	3	3	3	1	21	21	2	3	3	3
3rd	2	0	1	0	2	3	3	3	3	3	1	21	21	2	3	3	3

Table 17: Models with everything massive (including neutrinos) except the up quark, anomaly-free, and a minimal number of $UUUD$ operators (21). These occur in configuration 3. There are none in configuration 2, and a lot in configuration 1 but all of those have 27 $UUUD$ operators while these only have 21. There are also a number of models that have only 8 $UUUD$ operators, but these have one neutrino massless.

config.	H_u	H_d	p	A	B	\mathcal{U}_1	\mathcal{U}_2	\mathcal{U}_3	\mathcal{D}_1	\mathcal{D}_2	\mathcal{D}_3	B	L	Ups	Downs	Lept.	Neut.
1st	2	0	1	0	2	0	0	0	1	1	0	15	30	3	1	1	2
1st	2	0	1	0	2	0	0	0	3	3	0	15	30	3	1	1	2
1st	2	0	1	0	2	0	0	0	0	1	1	15	30	3	1	1	2
1st	2	0	1	0	2	0	0	0	1	0	1	15	30	3	1	1	2
1st	2	0	1	0	2	0	0	0	0	3	3	15	30	3	1	1	2
1st	2	0	1	0	2	0	0	0	3	0	3	15	30	3	1	1	2
2nd	0	2	0	0	2	0	0	0	1	1	0	15	24	1	1	1	2
2nd	0	2	0	0	2	0	0	0	3	3	0	15	24	1	1	1	2
2nd	1	1	1	3	3	1	1	1	0	0	1	12	6	0	1	0	2
2nd	1	1	1	3	3	1	1	1	2	2	1	18	42	0	2	2	2
2nd	2	0	0	2	0	2	2	2	1	1	2	0	9	2	1	0	2
2nd	2	0	0	2	0	2	2	2	3	3	2	0	9	2	1	0	2
2nd	3	3	1	1	1	3	3	3	0	0	3	12	6	0	1	0	2
2nd	3	3	1	1	1	3	3	3	2	2	3	18	42	0	2	2	2
3rd	2	0	0	0	2	0	0	0	1	1	0	15	24	1	1	1	2
3rd	2	0	0	0	2	0	0	0	3	3	0	15	24	1	1	1	2
3rd	3	3	1	3	3	1	1	1	0	0	1	12	6	0	1	0	2
3rd	3	3	1	3	3	1	1	1	2	2	1	18	42	0	2	2	2
3rd	0	2	0	2	0	2	2	2	1	1	2	0	9	2	1	0	2
3rd	0	2	0	2	0	2	2	2	3	3	2	0	9	2	1	0	2
3rd	1	1	1	1	1	3	3	3	0	0	3	12	6	0	1	0	2
3rd	1	1	1	1	1	3	3	3	2	2	3	18	42	0	2	2	2

Table 18: Anomaly-free models from all three configurations that have non-trivial mass matrices and no $UU\mathcal{D}$ operators. The remaining B and L violating operators can be forbidden by an anomaly-free Z_2 matter parity.

config.	H_u	H_d	A	B	p	\mathcal{U}_1	\mathcal{U}_2	\mathcal{U}_3	\mathcal{D}_1	\mathcal{D}_2	\mathcal{D}_3	B	L	Ups	Downs	Lept.	Neut.
1st	3	3	3	3	0	2	2	2	1	2	1	0	3	0	0	0	1
1st	0	2	2	0	1	2	2	2	1	2	1	0	3	0	0	0	2
1st	3	3	1	1	1	2	2	2	1	2	1	0	3	0	0	0	1
1st	3	3	3	3	0	2	2	2	2	1	1	0	3	0	0	0	1
1st	0	2	2	0	1	2	2	2	2	1	1	0	3	0	0	0	2
1st	3	3	1	1	1	2	2	2	2	1	1	0	3	0	0	0	1
1st	3	3	3	3	0	2	2	2	1	1	2	0	3	0	0	0	0
1st	0	2	2	0	1	2	2	2	1	1	2	0	3	0	0	0	2
1st	3	3	1	1	1	2	2	2	1	1	2	0	3	0	0	0	0
1st	1	1	1	1	0	2	2	2	3	3	2	0	3	0	0	0	0
1st	0	2	2	0	1	2	2	2	3	3	2	0	3	0	0	0	2
1st	1	1	3	3	1	2	2	2	3	3	2	0	3	0	0	0	0
1st	1	1	1	1	0	2	2	2	2	3	3	0	3	0	0	0	1
1st	0	2	2	0	1	2	2	2	2	3	3	0	3	0	0	0	2
1st	1	1	3	3	1	2	2	2	2	3	3	0	3	0	0	0	1
1st	1	1	1	1	0	2	2	2	3	2	3	0	3	0	0	0	1
1st	0	2	2	0	1	2	2	2	3	2	3	0	3	0	0	0	2
1st	1	1	3	3	1	2	2	2	3	2	3	0	3	0	0	0	1
2nd	2	0	2	0	0	2	2	2	1	1	2	0	9	2	1	0	2
2nd	3	3	3	3	0	2	2	2	1	1	2	0	9	0	1	0	1
2nd	3	3	1	1	1	2	2	2	1	1	2	0	9	0	1	0	1
2nd	1	1	1	1	0	2	2	2	3	3	2	0	9	0	1	0	1
2nd	2	0	2	0	0	2	2	2	3	3	2	0	9	2	1	0	2
2nd	1	1	3	3	1	2	2	2	3	3	2	0	9	0	1	0	1
3rd	0	2	2	0	0	2	2	2	1	1	2	0	9	2	1	0	2
3rd	1	1	3	3	0	2	2	2	1	1	2	0	9	0	1	0	1
3rd	1	1	1	1	1	2	2	2	1	1	2	0	9	0	1	0	1
3rd	0	2	2	0	0	2	2	2	3	3	2	0	9	2	1	0	2
3rd	3	3	1	1	0	2	2	2	3	3	2	0	9	0	1	0	1
3rd	3	3	3	3	1	2	2	2	3	3	2	0	9	0	1	0	1

Table 19: Anomaly-free models with no B violating operators.

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